Project Summary

Overview

The PI proposes a flexible algorithmic framework for using general multi-rate, method-of-lines solvers on adaptively refined Cartesian meshes. This framework will enhance the scalability, performance and general usability of finite volume schemes for solving partial differential equations on adaptively refined meshes.

Intellectual merit of the proposed activity

Single step, single stage multi-rate schemes are routinely used for solving PDEs on adaptively refined meshes. However, such methods are usually limited to second order accuracy or may suffer from operator splitting errors. Higher order temporal discretizations involving multiple stages or complex coupling strategies are considerably more difficult to incorporate into existing software frameworks for adaptive mesh refinement.

To address this shortcoming, the PI proposes an algorithmic framework for flexible time discretization in the Cartesian grid adaptive setting. One nontrivial technical barrier to overcome is to design an abstract framework that is descriptive enough to include complex coupling strategies used to avoid splitting errors as well as higher order multi-stage discretizations. For multi-stage methods, the PI anticipates providing functionality that effectively provides a right-hand side vector typically required by ODE solvers. This will require designing an efficient, scalable data pipeline that provides a vectorized view of spatial data. Emphasis will be focused on explicit multi-stage Runge-Kutta methods for hyperbolic and parabolic equations.

Targeted applications of this work include the application of the theory of multi-rate methods for ODEs to the method-of-lines setting, the implementation of multi-rate, explicit Runge-Kutta-Chebyshev methods for reaction diffusion equations, and a demonstration of the effectiveness of the proposed framework on the important hazards modeling problem. The PI will incorporate a legacy code (Ash3D, USGS) for simulating the transport of airborne volcanic ash into the adaptive, multi-rate setting. With adaptivity, the simulation can be extended to the sphere at reasonable computational cost.

This work in this proposal will build upon the adaptive, multi-block, parallel software platform ForestClaw, developed by the PI and collaborator C. Burstedde (Univ. of Bonn, Germany).

Broader impact of the proposed activity

A major goal of the proposed research is to provide a general framework for temporal discretizations in the adaptive, multi-rate setting that is as convenient to use as that currently available for spatial discretizations. This will lower the threshold for entry into the adaptive, parallel world and encourage application developers to consider adaptive methods. By attracting academic researchers to this challenging environment, the PI and collaborators expect to broaden the applicability of adaptive methods, accelerate the deployment of large scale applications to the petascale and beyond, and ultimately to enhance scientific discovery.

The PI will continue her commitment to education in computational science and engineering by advising and mentoring graduate and undergraduate students, organizing a student chapter of SIAM, and eagerly serving as a role model and attract female students to the field of computational science and mathematics.