Project Summary

Overview

The PI proposes to implement a direct solver for elliptic problems on adaptively refined quadtree meshes. By using a direct solver, we will retain the advantages of matrix-free, iterative methods such as multigrid and Krylov subspace solvers, while overcoming the shortcomings of these methods. A targeted application is the solution to the Serre-Green Naghdi equations for modeling dispersive corrections to the shallow water wave equations.

Intellectual merit of the proposed activity

Effective and popular methods for solving elliptic problems on adaptively refined meshes include geometric multigrid and Krylov subspace solvers, and, as problems become more challenging, algebraic multigrid. However, there has been little appetite for using direct solvers on dynamically adaptive meshes, in part because direct methods typically required matrix assembly and second, expensive factorizations must be rebuilt whenever the adaptive mesh changes.

Recent developments in direct solver technology have overcome some of the above issues, while retaining the advantages of multigrid and iterative solvers. The Hierarchical Poincaré Steklov (HPS) solver, developed by Gillman and Martinsson is one such solver that (1) has the ease of use of matrix-free methods, (2) can solve nearby systems quickly, (3) has optimal $O(N)$ efficiency, and (4) provides parameters that can be tuned to reduce computational cost in proportion to accuracy requirements.

The PI will modify this solver for use with second order, finite volume schemes and implement the solver in ForestClaw, a patch-based adaptive quadtree code under development by the PI. While it has been shown that there are essentially no barriers to using the HPS solver on a serial platform with adaptive meshes, results on parallel efficiency has yet to be reported. Two main challenges to overcome are how to merge successive Dirichlet-to-Neumann maps across processor boundaries, and how to incrementally update the factorization for dynamically evolving meshes.

The solver will be used to solve the Serre-Green Naghdi equations and include correction terms to the GeoClaw extension of ForestClaw. This capability will give GeoClaw users the ability to model dispersive terms in geophysical flows, including tsunamis and debris flows.

Broader impact of the proposed activity

A major goal of the proposed research is to establish an approach to incorporating direct solver technology into adaptive mesh frameworks. This should point the way to new approaches to handling elliptic terms in challenging exascale environments, and accelerate the deployment of large scale applications to the petascale and beyond, and the ultimately enhance scientific discovery.

The PI will continue her commitment to education in computational science and engineering by advising and mentoring graduate and undergraduate students, promoting the newly developed Minor degree in Computational Science and Engineering at BSU, and eagerly serving as a role model and attract female students to the field of computational science and mathematics.