Algorithmic components of ForestClaw adaptive mesh library : Multi-rate time stepping

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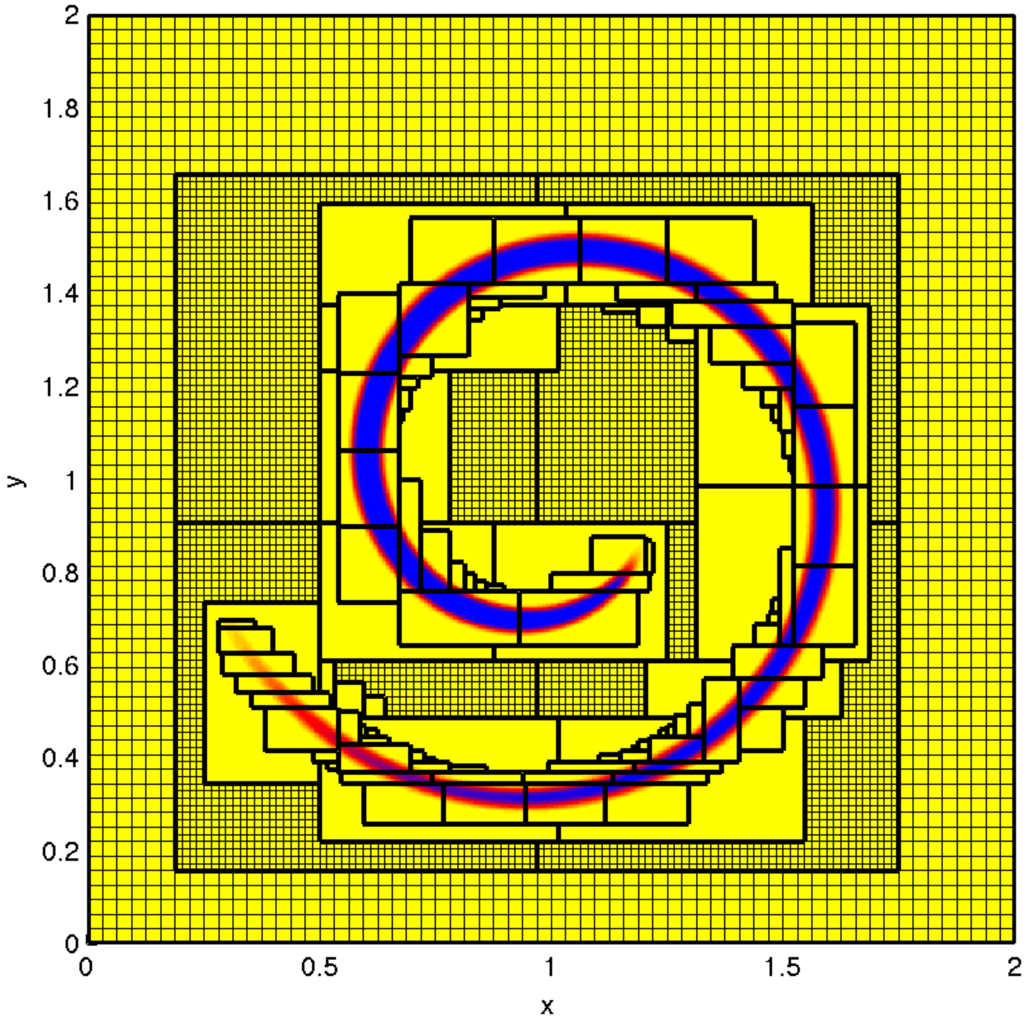
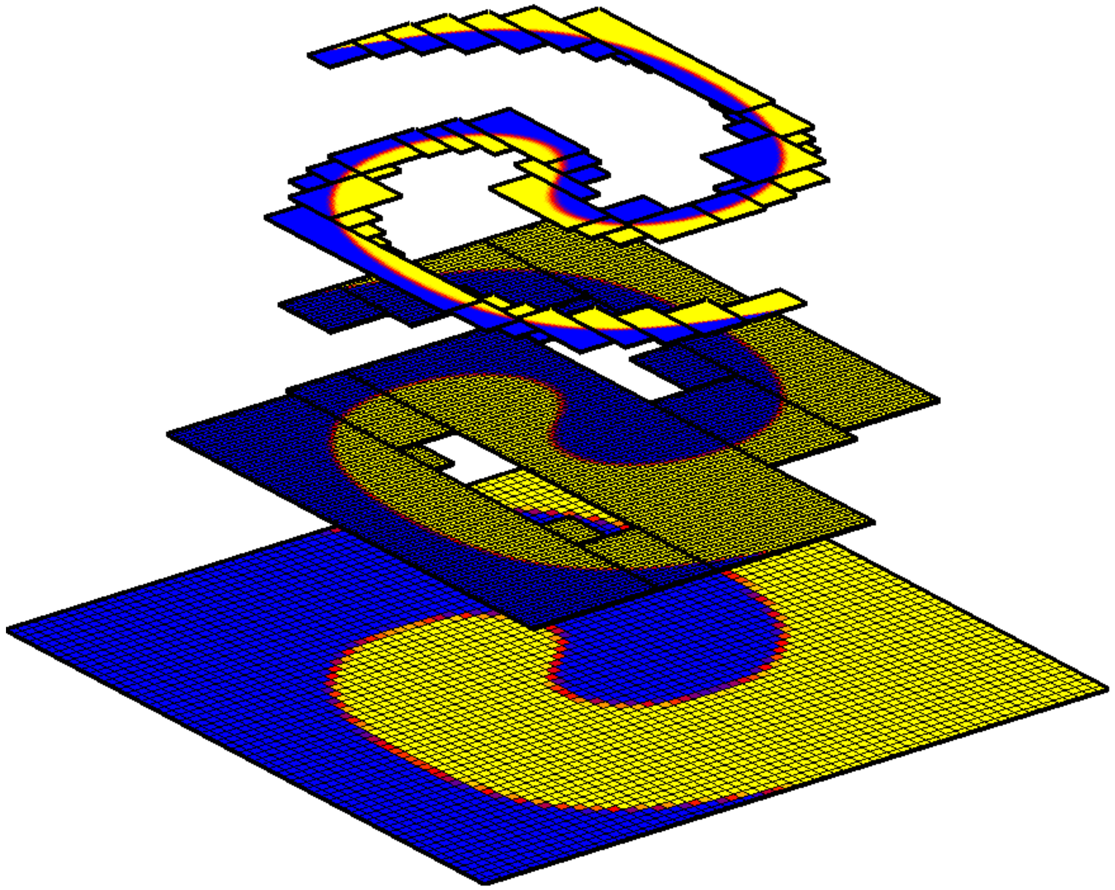
SIAM Parallel Processing
April 12-15, 2016
Universite Pierre et Marie Curie
Paris, France

Adaptive Mesh Refinement (AMR)

Overlapping patch-based AMR

Original approach (Berger, 1984)





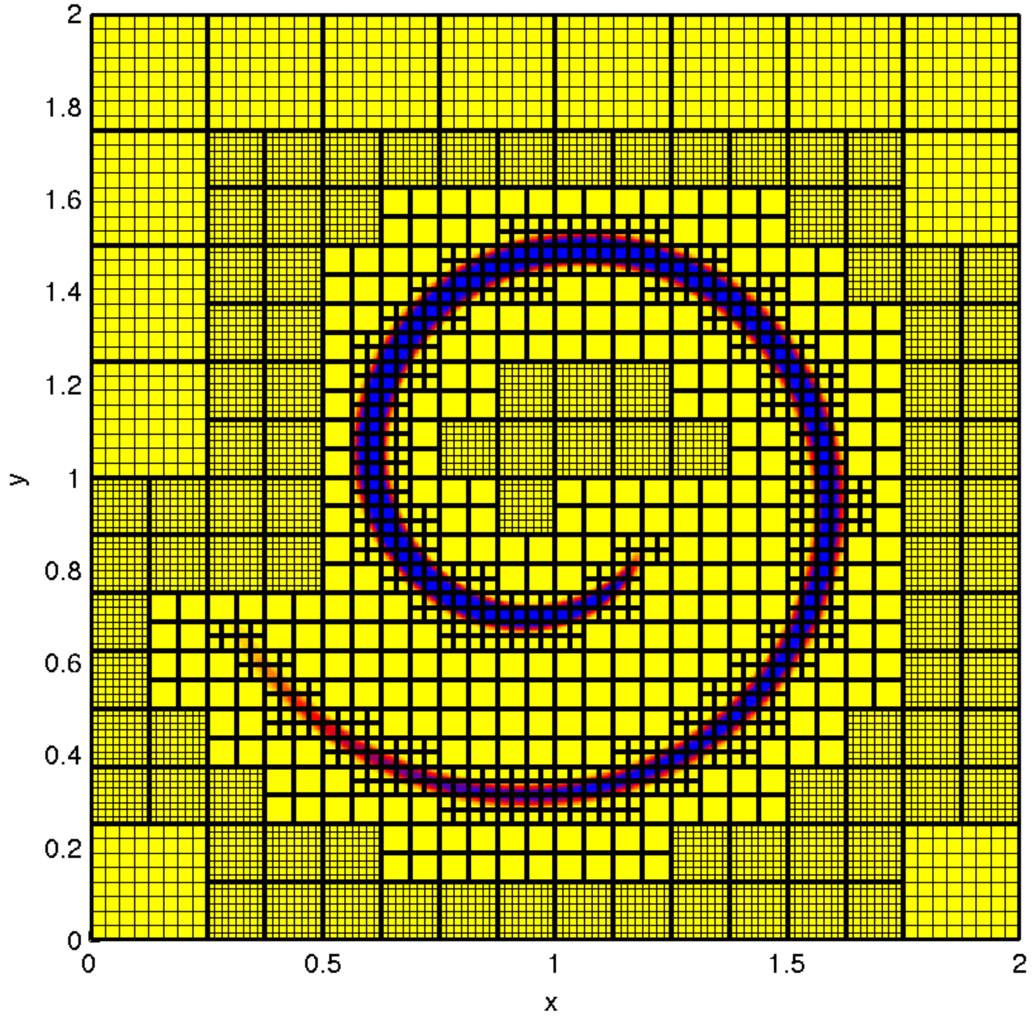
Chombo (LBL), AMRClaw (UW, NYU) , **Boxlib* (LBL)**, SAMRAI (LLNL), AMROC (S. Hampton)

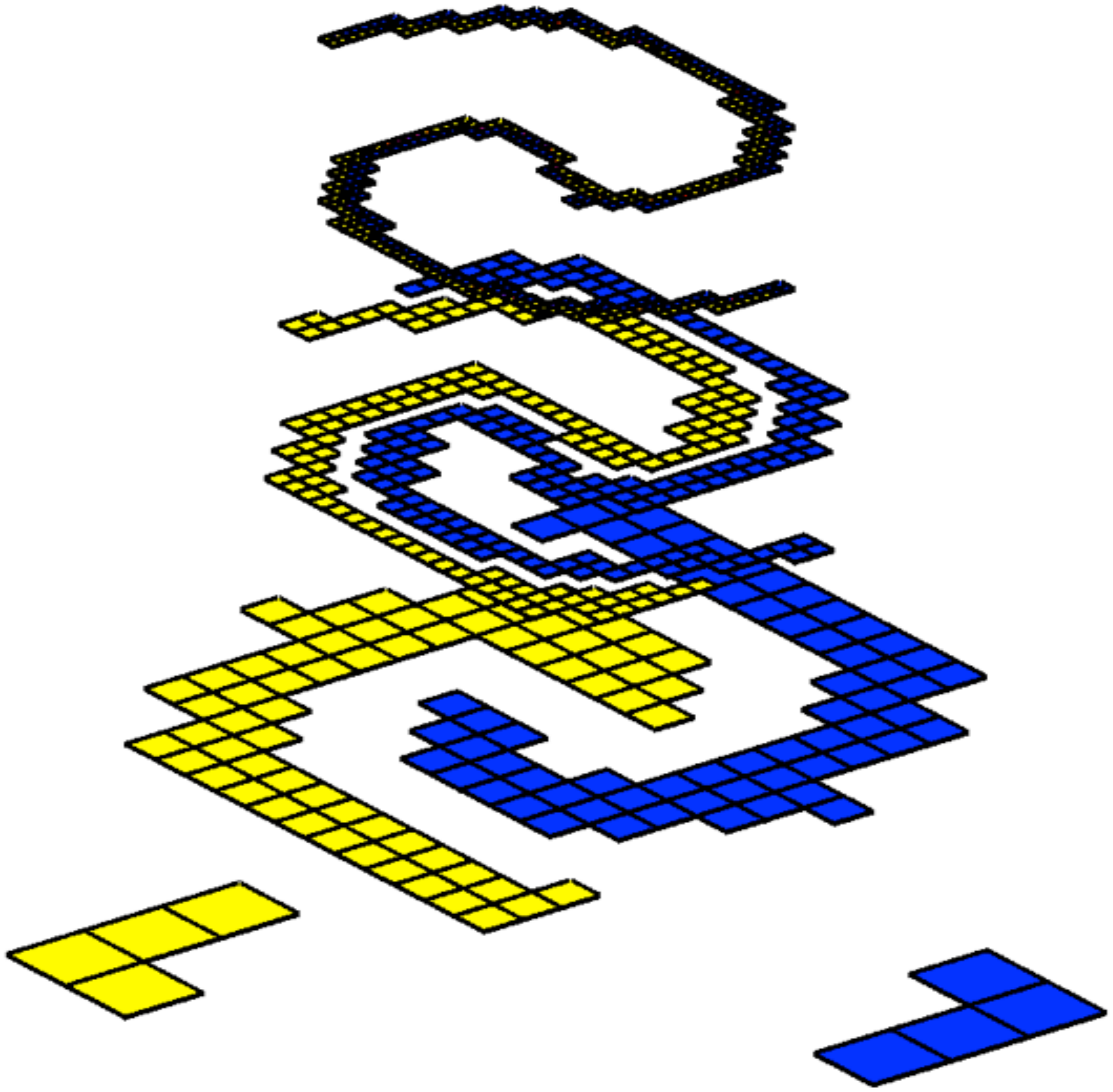
**See IP4 : "Next Generation AMR (A. Almgren, Thursday 8:30)*

Adaptive Mesh Refinement (AMR)

Quadtree/Octree based AMR

Quad-tree approach





p4est (U. Bonn), PARAMESH (U. Chicago), **ForestClaw**, Gerris (Paris VI), Racoon II (U. Bochum), RAMSES (U Zurich), Nirvana (Potsdam), "Building Cubes" (Tohoku)

Skepticism about AMR

There remains skepticism about how effective adaptive mesh refinement (AMR) can be

- Coarse/fine boundaries with abrupt resolution changes are regarded with suspicion,
- Lack of good refinement criteria dampens enthusiasm for trying out AMR,
- Not obvious how to extend sophisticated numerical algorithms and applications to the adaptive setting,

When AMR *is* used,

- **Multi-rate time stepping is not always used,**
- The goals are often modest :"Do no harm!"
- Grids are often only static; not dynamically refined.

ForestClaw Project

A parallel, adaptive library for logically Cartesian, mapped, multi-block domains

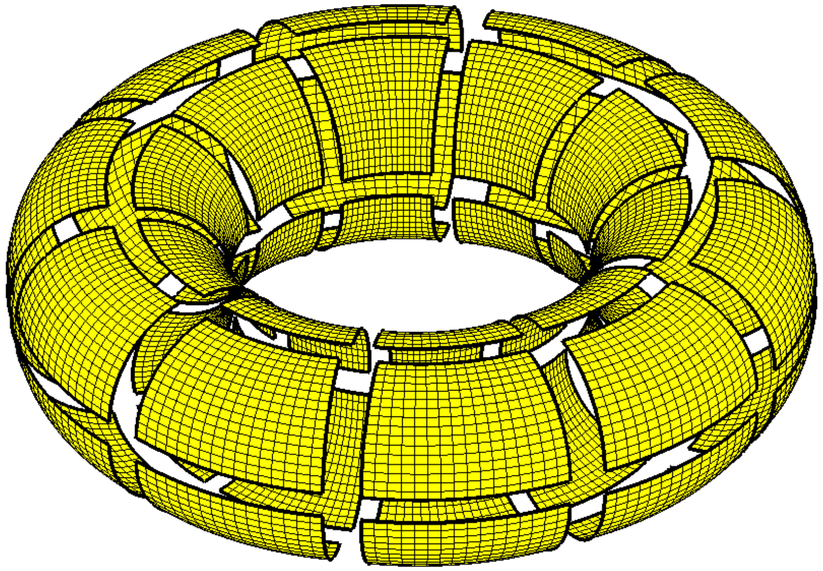
Features of ForestClaw include :

- Uses the **highly scalable p4est** dynamic grid management library (C. Burstedde, Univ. of Bonn, Germany)
- Each leaf of the quadtree contains a fixed, uniform grid,
- **Optional multi-rate time stepping strategy,**
- Has **mapped, multi-block** capabilities, (cubed-sphere, for example) to allow for flexibility in physical domains,
- Modular design gives user flexibility in including several solvers and packages.
- Uses essentially the same algorithmic components as patch-based AMR

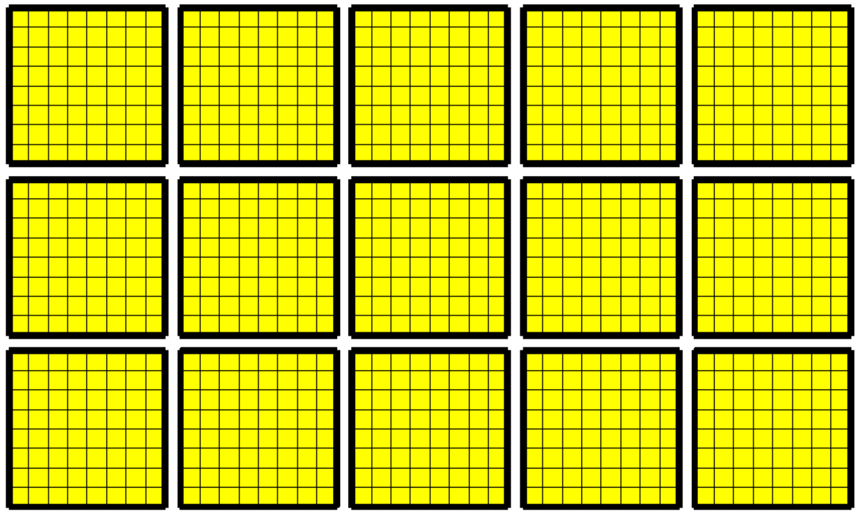
ForestClaw development supported by the National Science Foundation

www.forestclaw.org

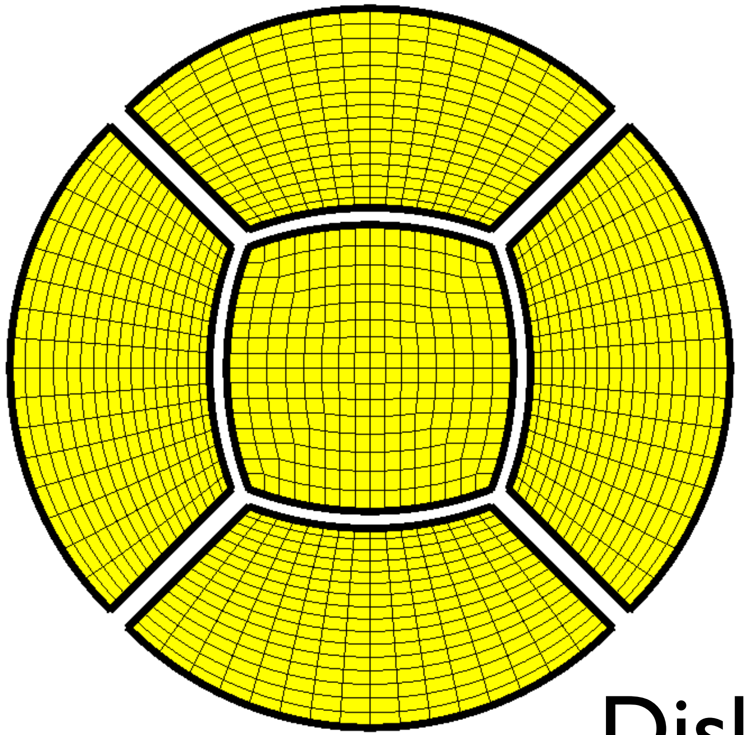
Mapped multi-block domains



Torus

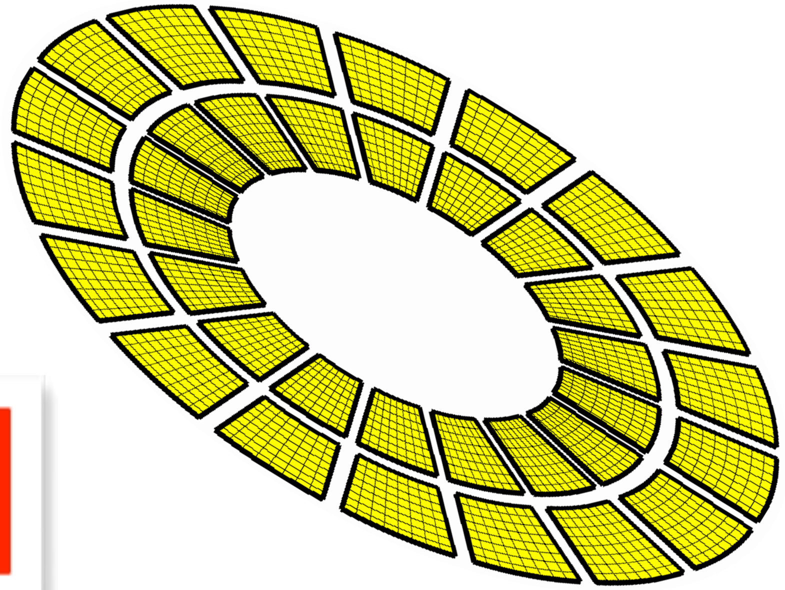


“Brick domain”



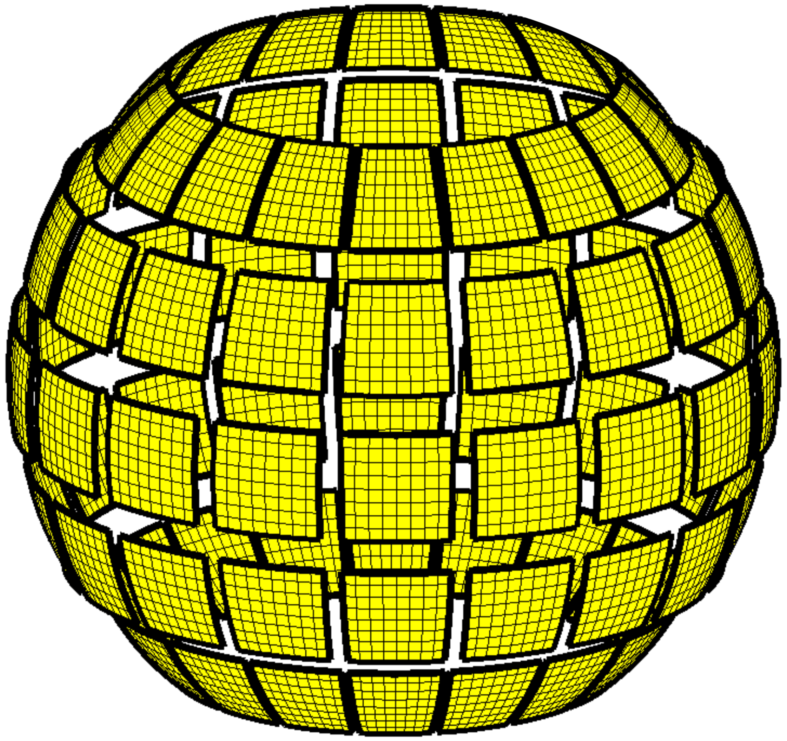
Disk

Each block is a quadtree

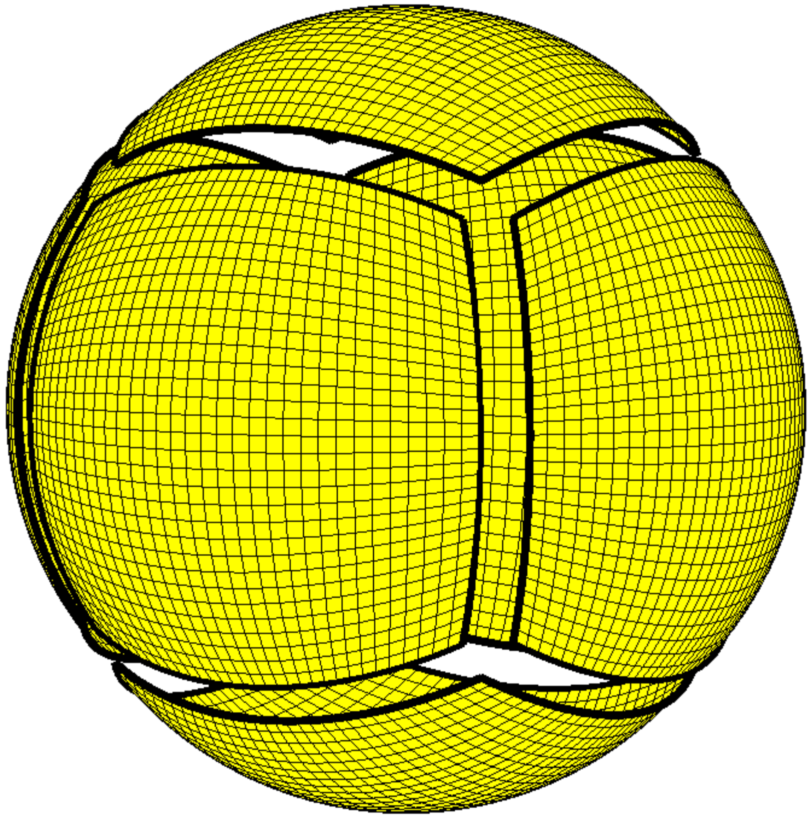


Annulus

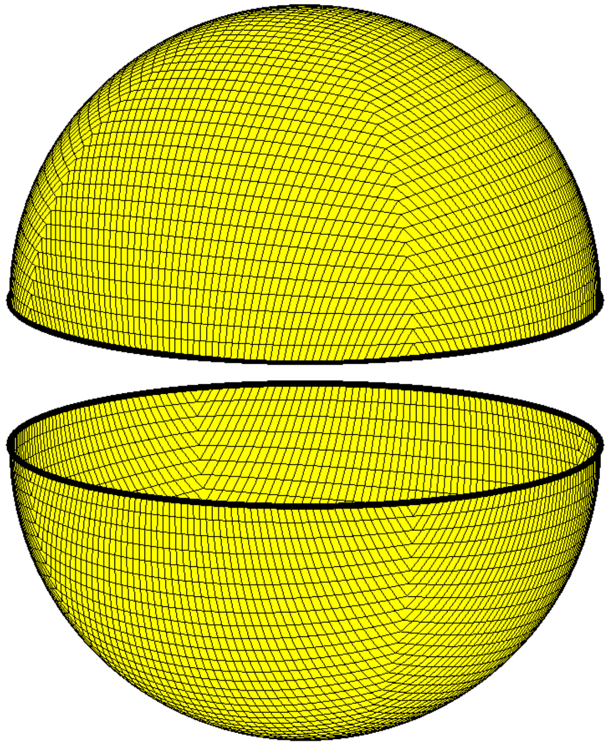
Mapped multi-block domains



Spherical coordinates



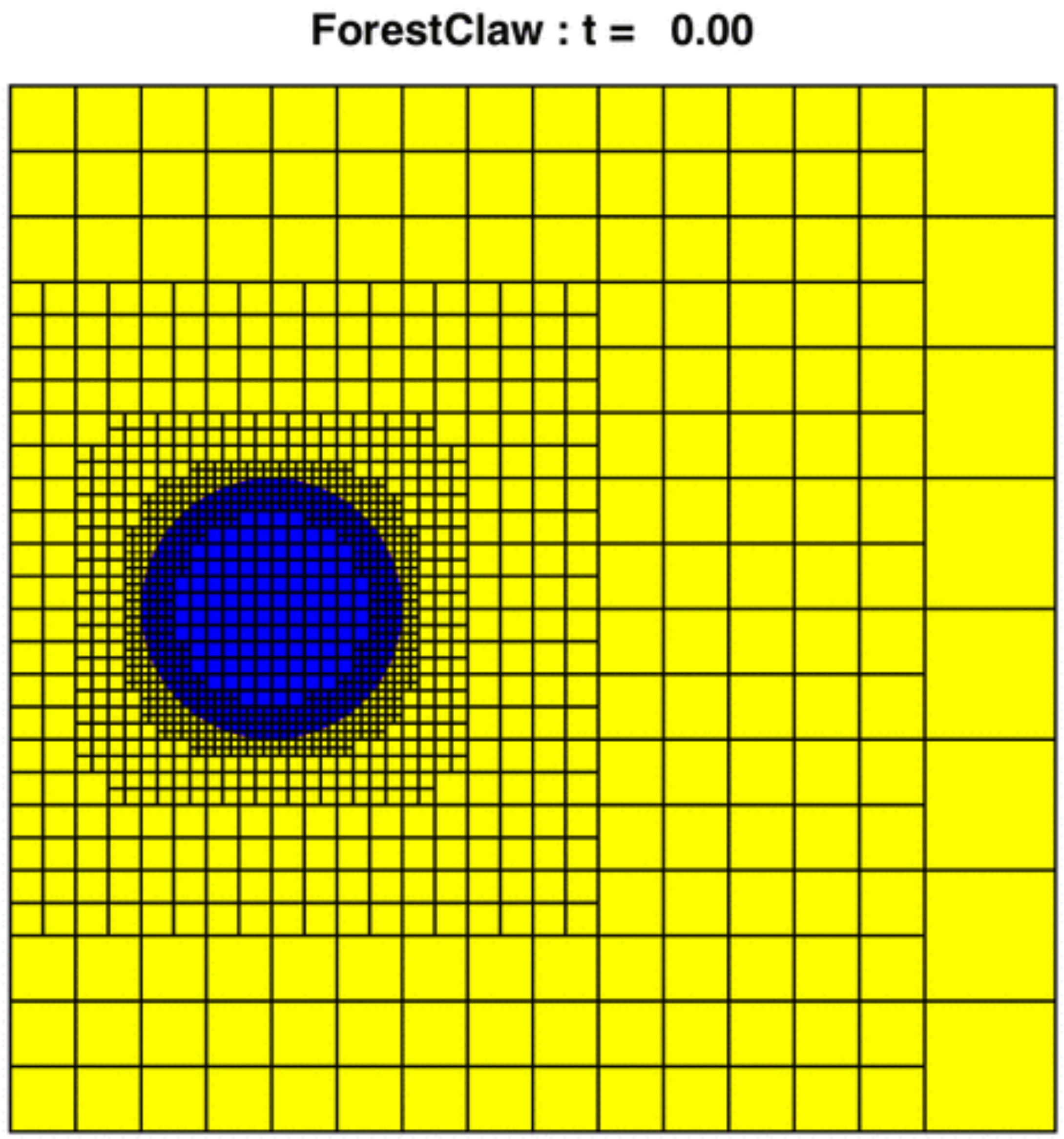
Cubed sphere



“Pillow grid” (with C. Helzel, R. LeVeque, SIAM Review, 2009)

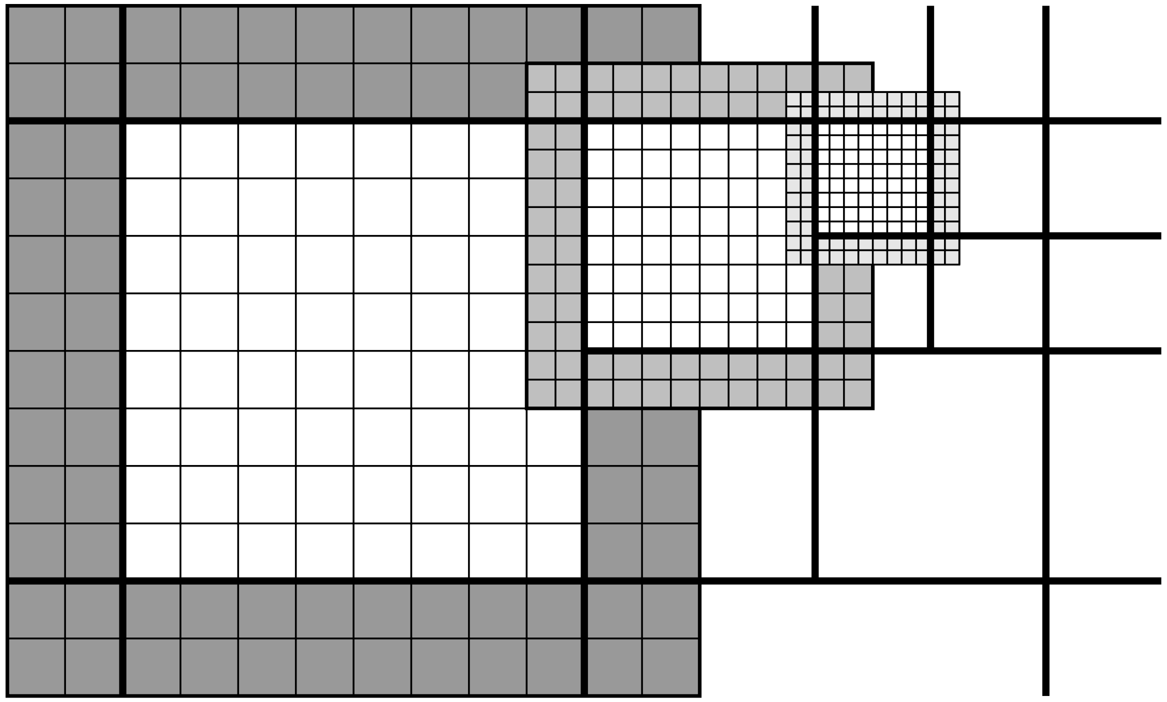
Each block is a quadtree

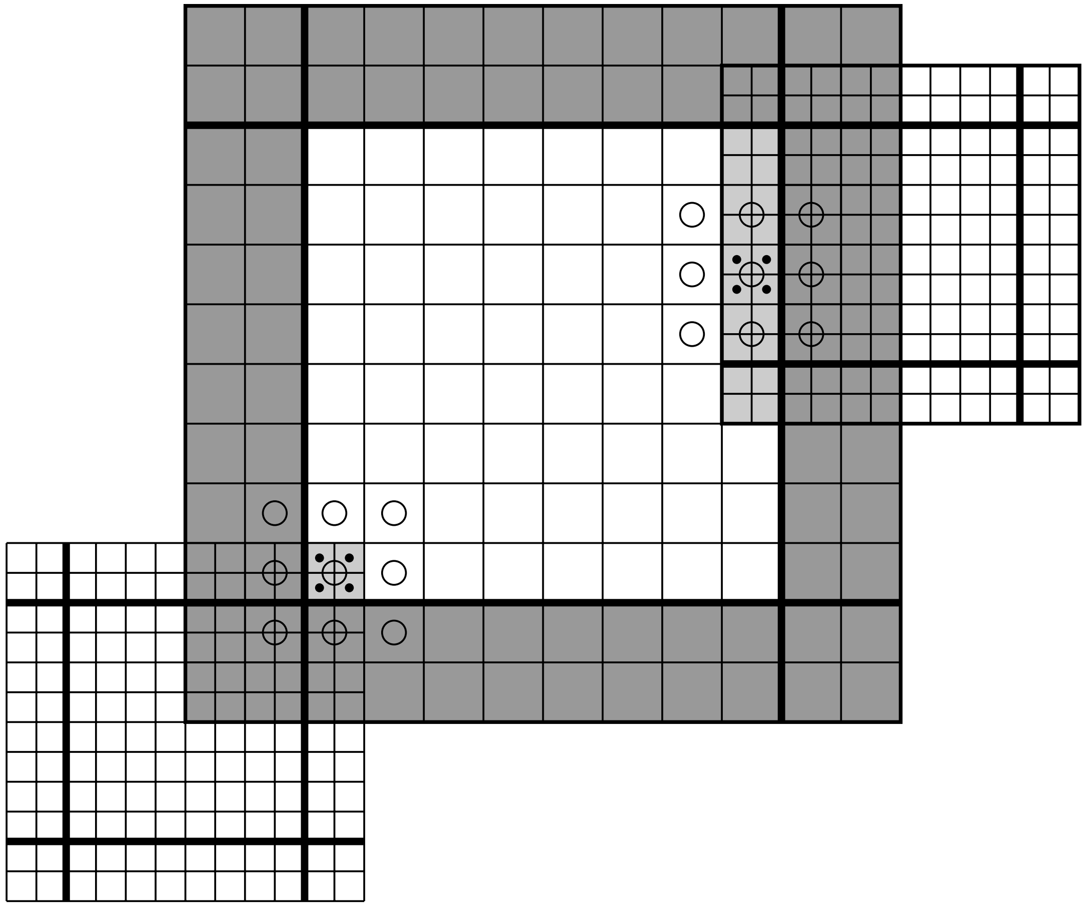
Filament



Tracer flow in a prescribed velocity field

Filling ghost cells





Each grid (or "leaf", in p4est terminology) has one of more layers of ghost cells used for communication between grids

Ghost cells are filled by copying from same-size neighbors, averaging from fine grid neighbors or interpolation to fine grid neighbors.

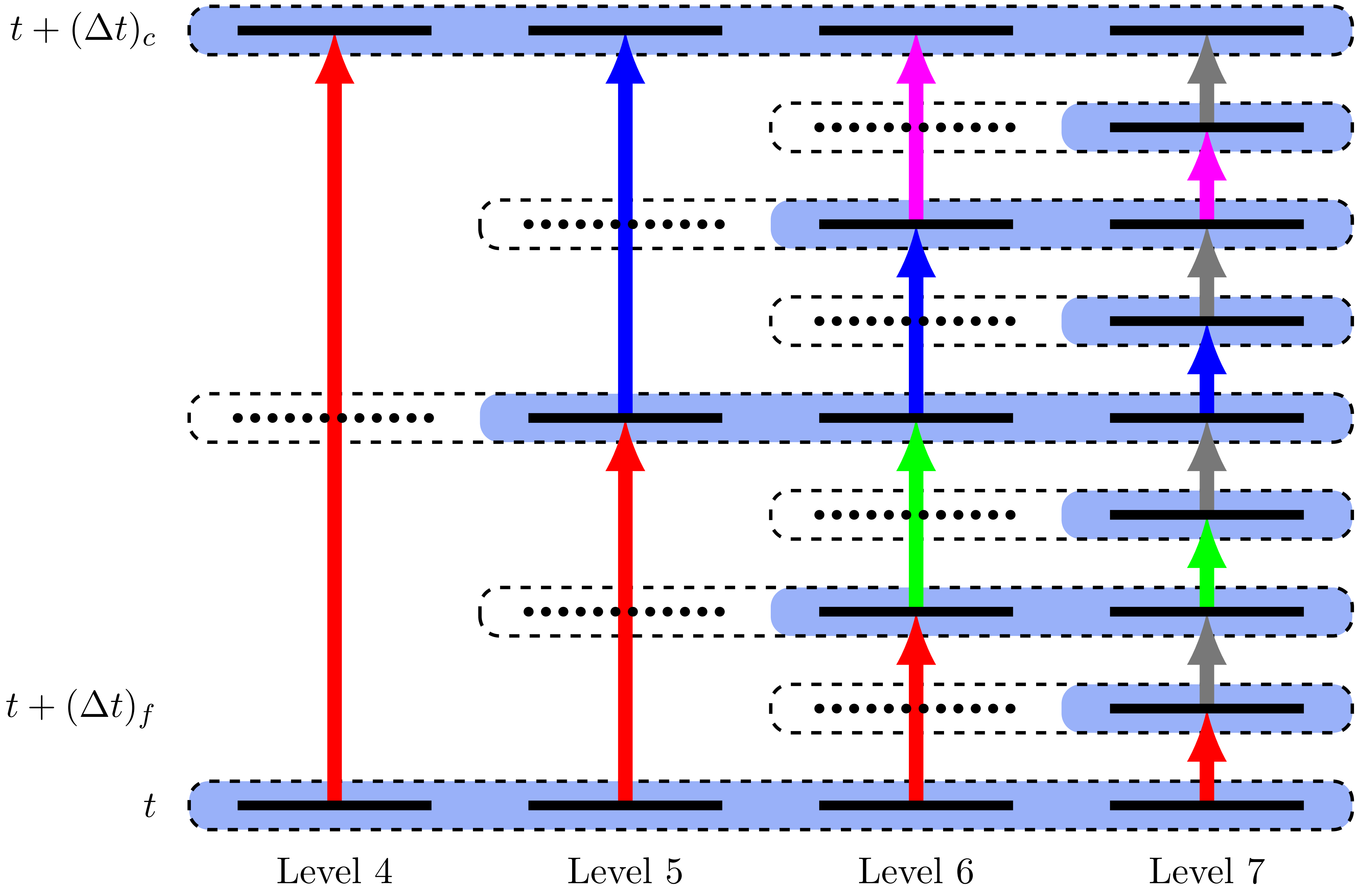
How are ghost cells filled?

Copy

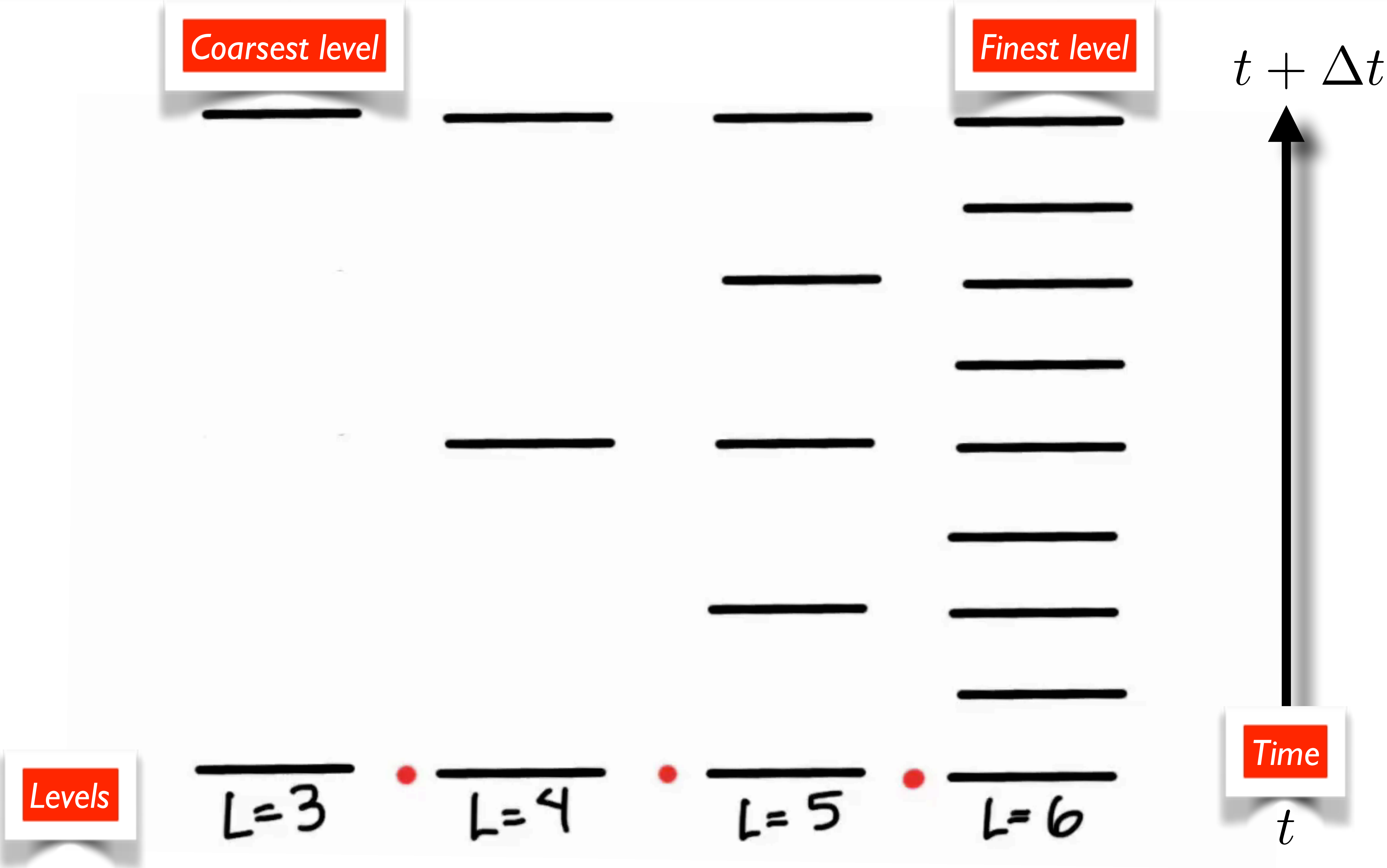
Average

Interpolate

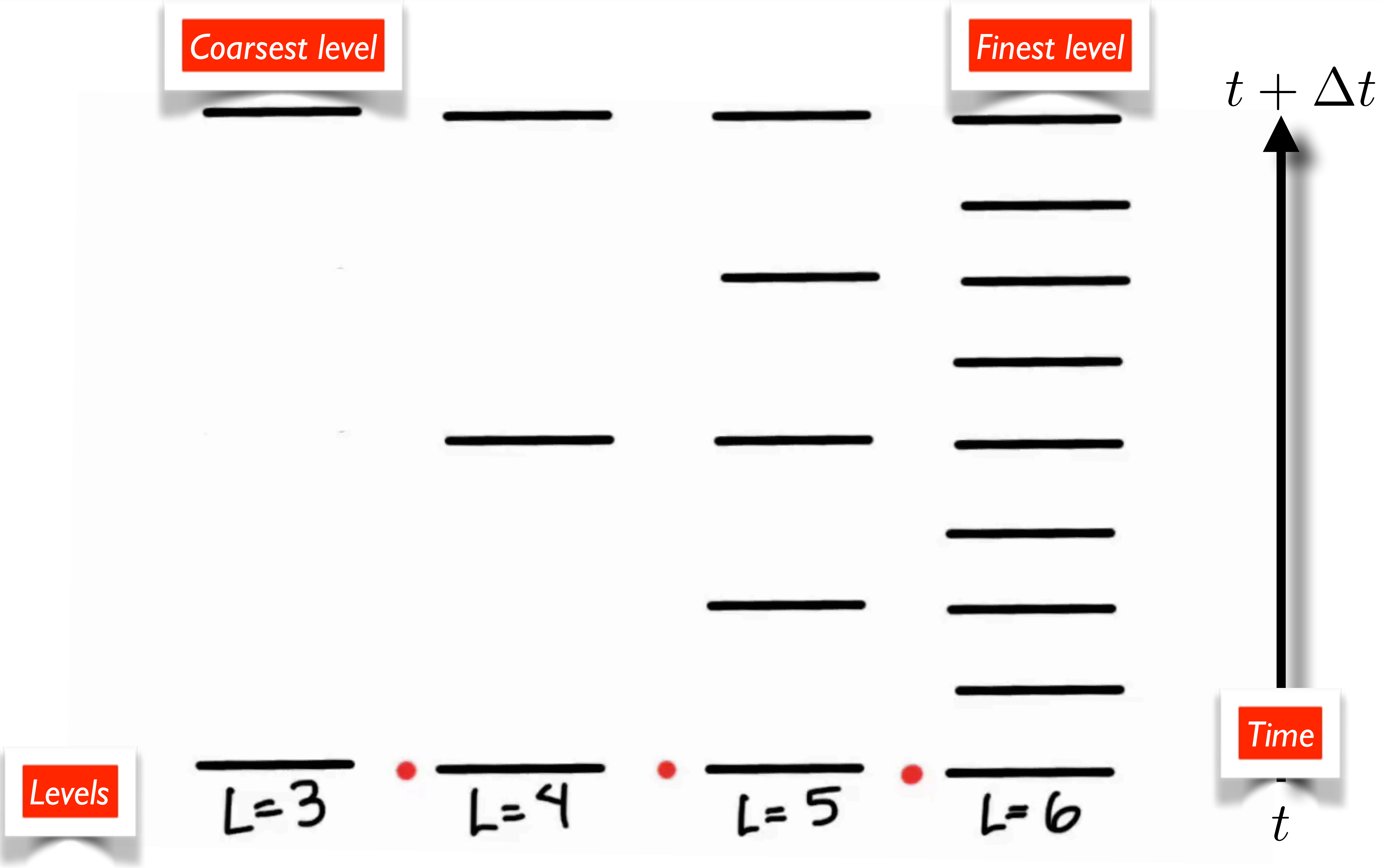
Multirate time stepping



Multi-rate time stepping



Parallel multirate time stepping



Multirate algorithm

```
Require: Grids at all levels at time $t$ must have valid ghost cells values.
  for $k = 1$ to $2^{\ell_{max}-\ell_{min}}$ do
    ADVANCE_SOLUTION($\ell_{max}, (\Delta t)_{\ell_{max}}$)
    if multirate then
      if $k < 2^{\ell_{max}-\ell_{min}}$ then
        Find largest integer $p \geq 0$ such that $2^p$ divides $k$.
        $\ell_{time} = \ell_{max} - p - 1$
        UPDATE_GHOST($\ell_{time} + 1$)
      end if
    else
      UPDATE_GHOST($\ell_{min}$)
    end if
  end for
  UPDATE_GHOST($\ell_{min}$).
```

Advance solution on finest level

Multirate

Intermediate synchronization

Global time stepping

```
procedure ADVANCE_SOLUTION(level $= \ell$, dt_stable $= \Delta t$)
  for all grids $g$ on level $\ell$ do
    Update solution $Q^{n+1} = Q^n + \Delta t\ F(Q^n, t_n)$.
  end for
  if $\ell > \ell_{min}$ then
    if multirate then
      if levels $\ell$ and $\ell - 1$ are time synchronized then
        ADVANCE_SOLUTION($\ell - 1, 2\Delta t$)
        TIME_INTERPOLATE($\ell - 1, t + 2\Delta t$)
      end if
    else
      ADVANCE_SOLUTION($\ell - 1, \Delta t$)
    end if
  end if
end procedure
```

Recursive advance, followed by a time interpolation

Multi-rate time stepping?

Does this buy us anything?

- Fewer ghost cell exchanges, since not all levels exchange data at each time step,
- Fewer parallel communications
- Fewer grids to advance.
- Improved accuracy if spatial error depends on time step size

Reduced wall clock time!

Improved accuracy!

Is multirate time stepping worth it?

Assuming an equal number of grids at each level :

$$(1 + 2 + 4 + 8 + \ldots + 2^{\ell_{max} - \ell_{min}}) = (2^{\ell_{max} - \ell_{min} + 1} - 1)$$

verses

$$2^{\ell_{max} - \ell_{min}}(\ell_{max} - \ell_{min} + 1)$$

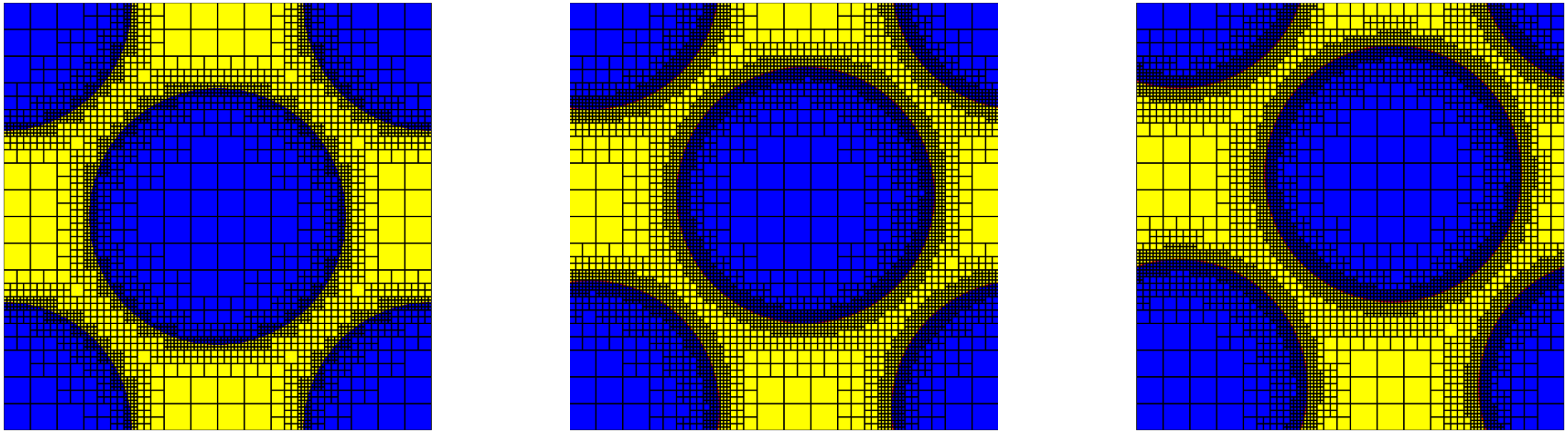
on a uniformly refined grid, for a speed-up of

$$\begin{aligned} \frac{\text{multi rate}}{\text{global time step}} &= \frac{2}{\ell_{max} - \ell_{max} + 1} - \frac{1}{2^{\ell_{max} - \ell_{min}}(\ell_{max} - \ell_{min} + 1)} \\ &\approx \frac{2}{\ell_{max} - \ell_{min} + 1}, \qquad \text{for large number of levels} \end{aligned}$$

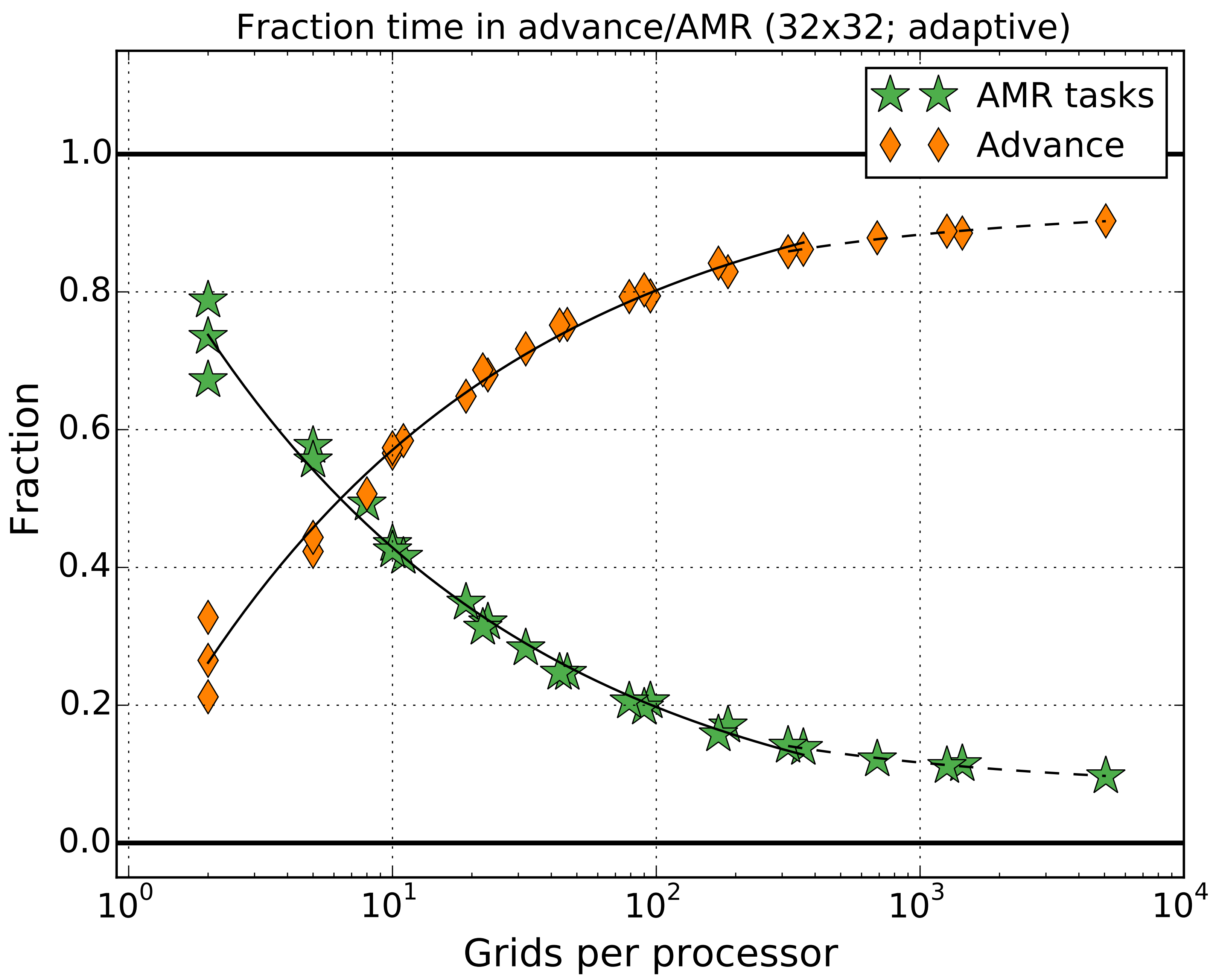
A multi-rate strategy also requires fewer parallel exchanges

Parallel Performance

- Scalar advection in a constant, periodic flow
- Uniform, replicated adaptive, and adaptive scenarios
- Fixed, global time step (160 steps total)
- Regrid every coarse grid time step (20 regrid steps)
- Four levels of refinement (levels 4-7)
- MPI processes only; no threading or tuning, and no attention paid to processor topology
- JUQUEEN BlueGene/Q (Juelich Supercomputing Center, Germany)

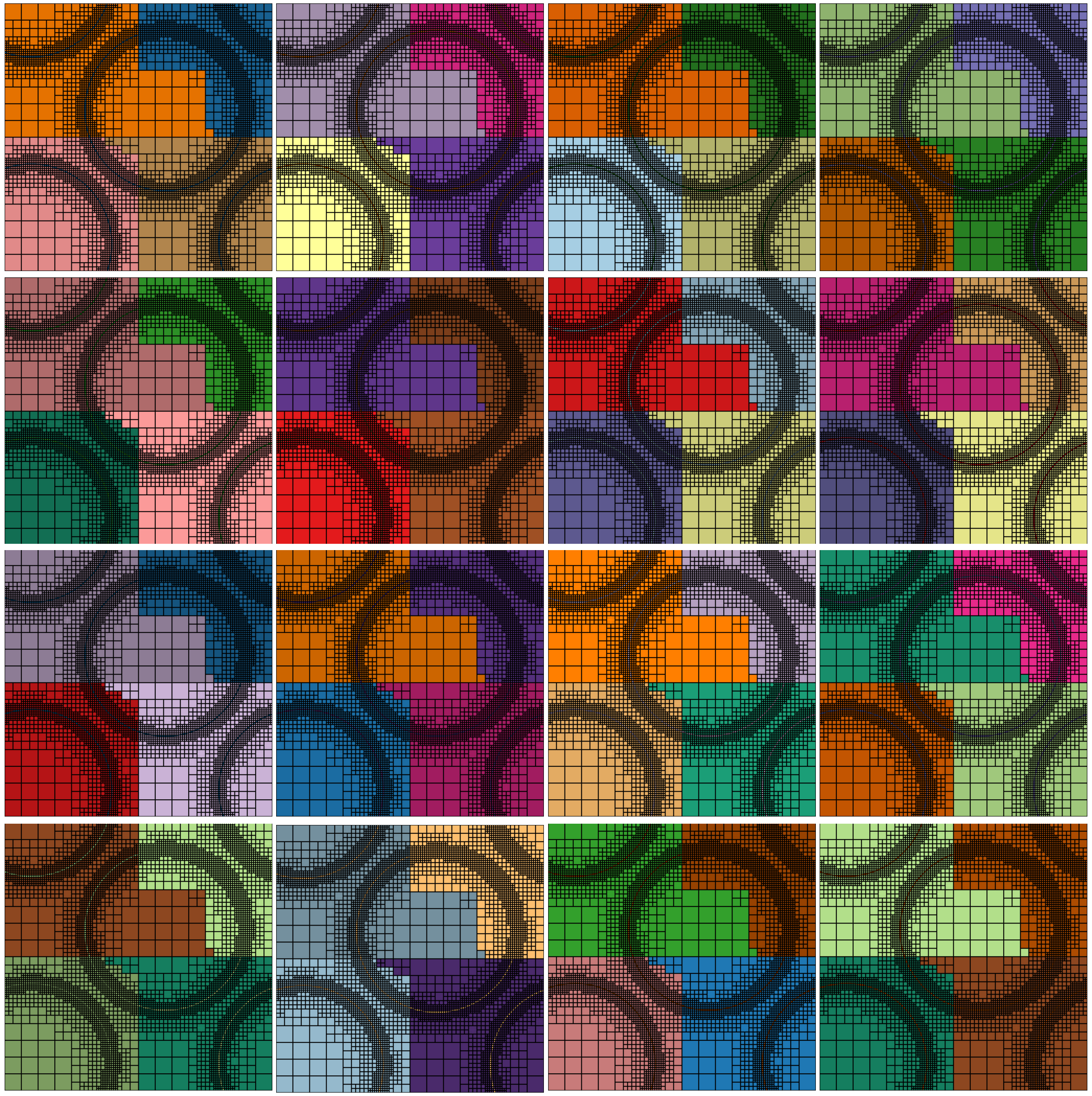


Cost of AMR for this problem



AMR tasks/Solving problem as a function of granularity

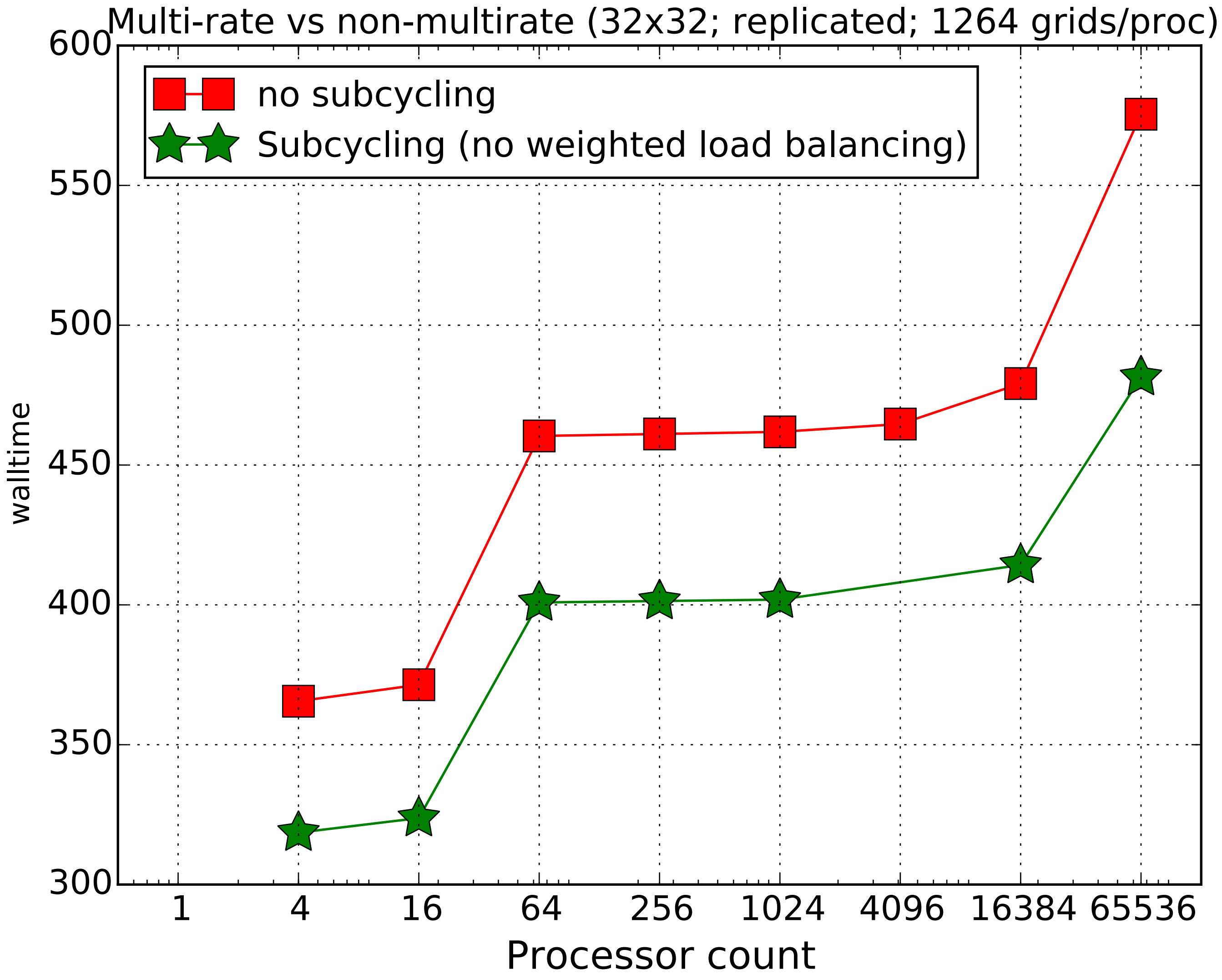
Replicated problem



64 processes
(four per brick)

Fixed resolution

Walltime - 32x32 (replicated)



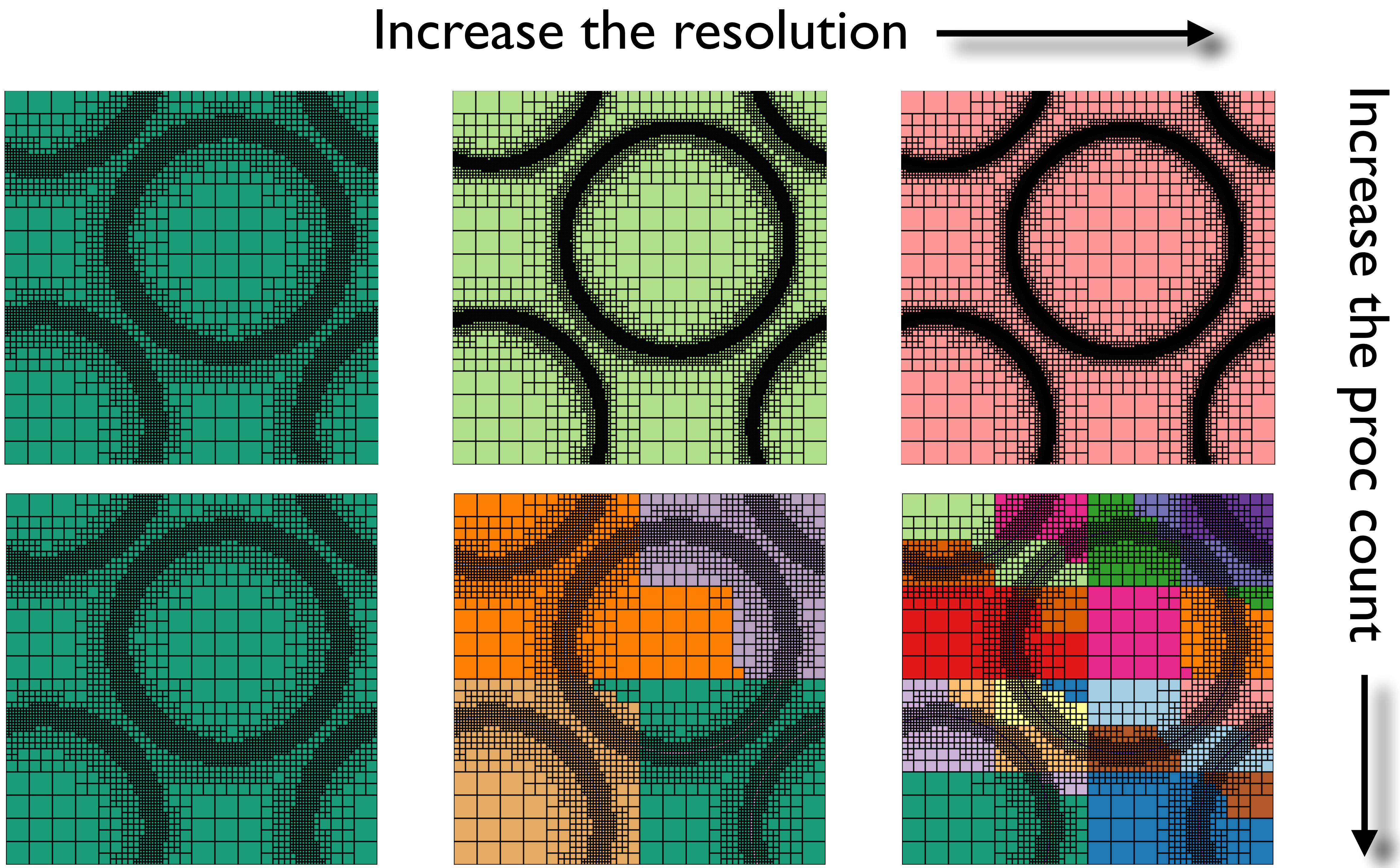
non-multirate

multirate

Non-multirate steps : 202,396
Multirate steps : 172,427
Difference : : 15%

Actual speed-up : 13%

Single adaptive problem

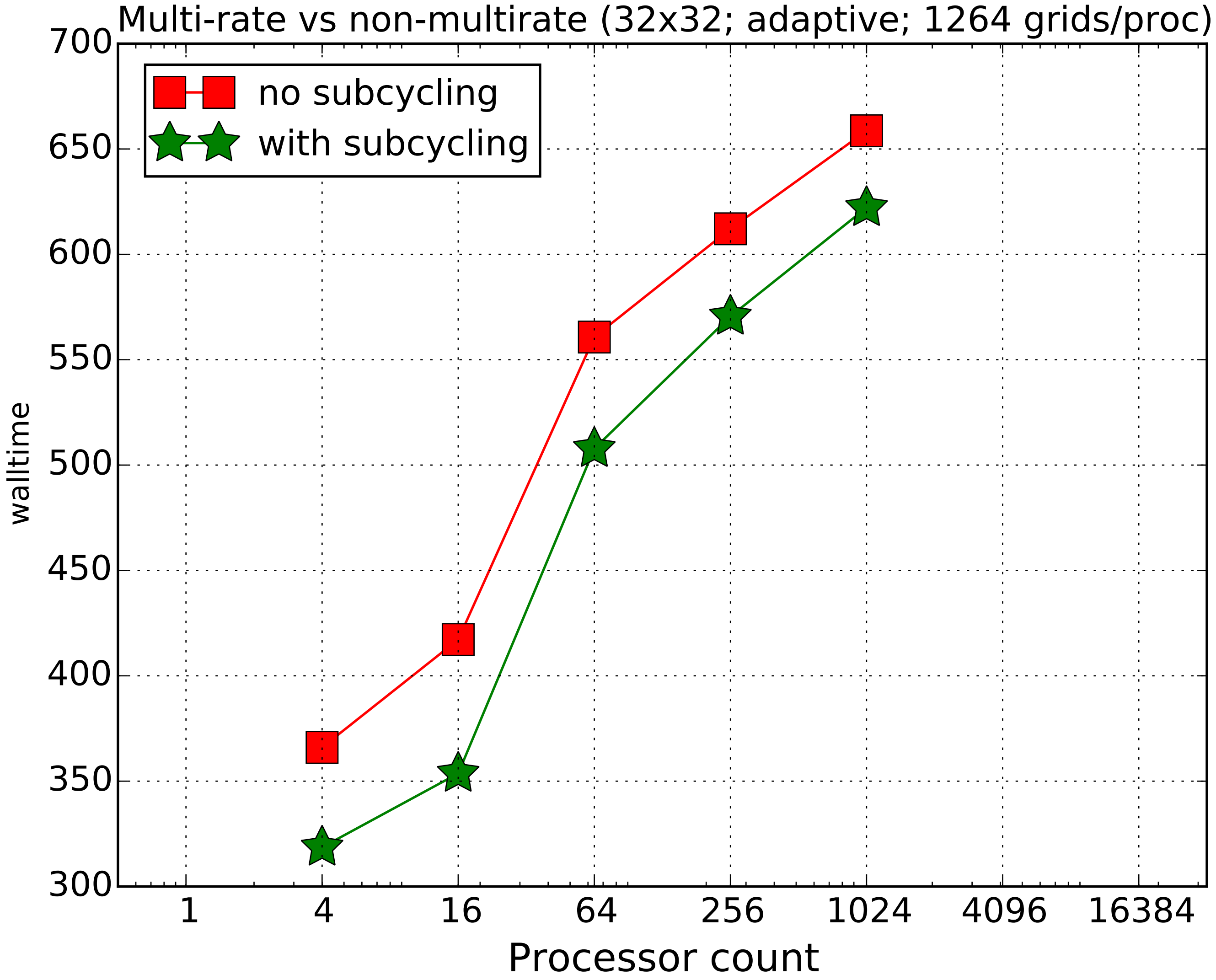


Increase the resolution

Increase the proc count

Walltime - 32x32 (single)

Cannot do better than this



94% (76%)

93% (78%)

82% (79%)

85% (81%)

87% (85%)

Wall clock time savings (%) (Advance steps savings %)

(Smaller % is better)

Conclusions

Is multi-rate worth it?

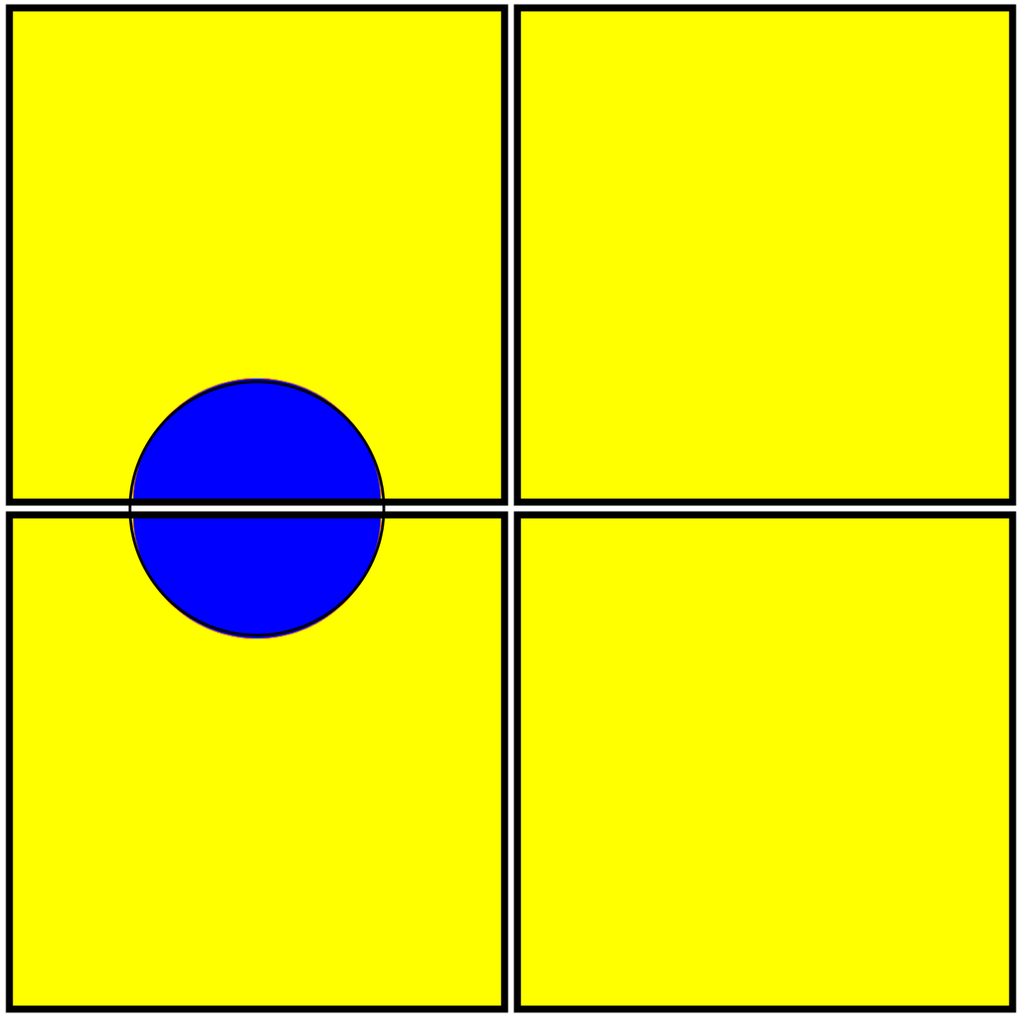
- Absolutely yes. It doesn't have to be difficult to include, and will generally be faster.
- Essential for many problems
- Savings generally depend on how much of the domain is covered in fine grids
- If spatial accuracy depends on the time step size, then multi-rate schemes should be more accurate

Caveat

- Mixed equations (hyperbolic, parabolic, elliptic) present challenge for multi-rate schemes
- Additional work may be needed to maintain conservation, accuracy at coarse/fine boundaries.

Subdivide the domain

6997.1s (1.9hr)



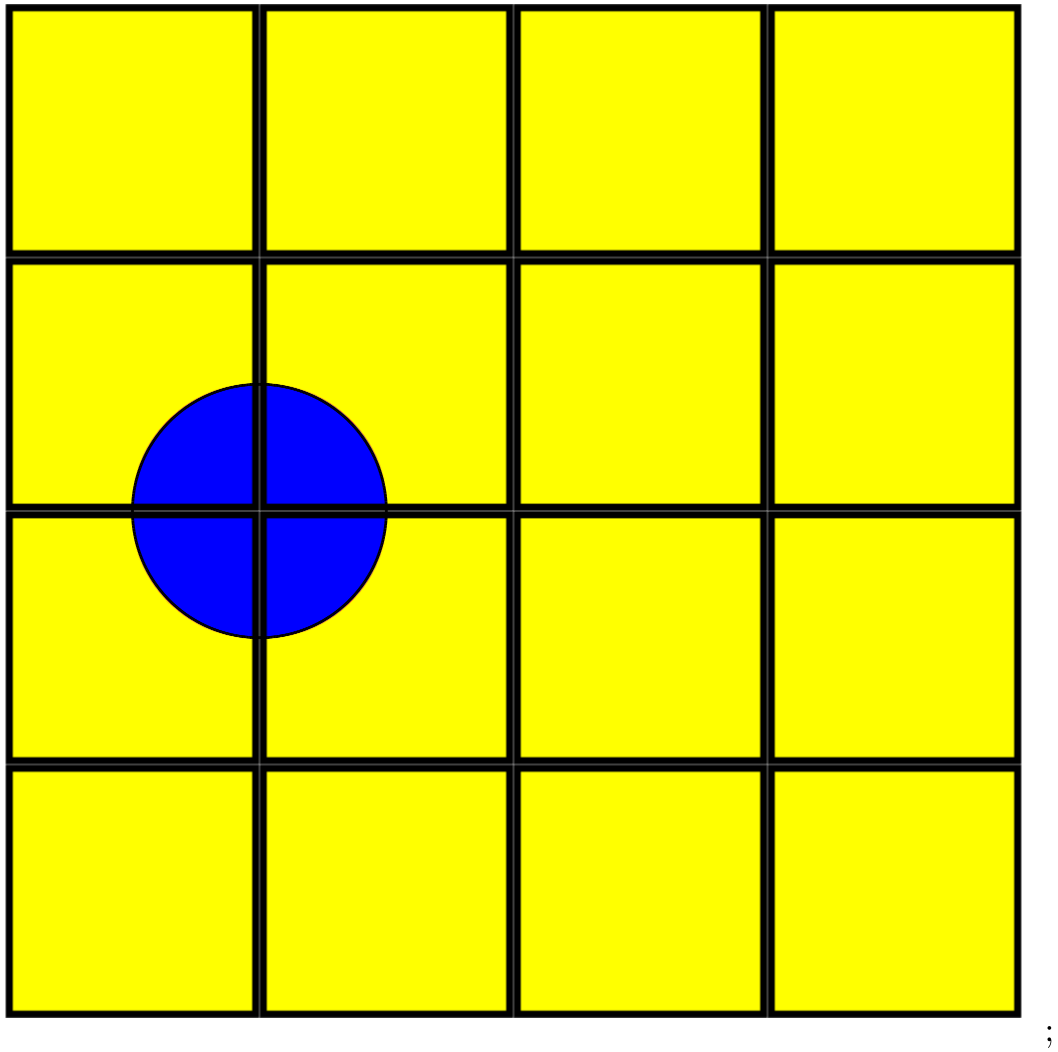
;

2×2 array of 512×512 grids

This will improve cache performance enormously

Subdivide the domain

4131.7s (69min)

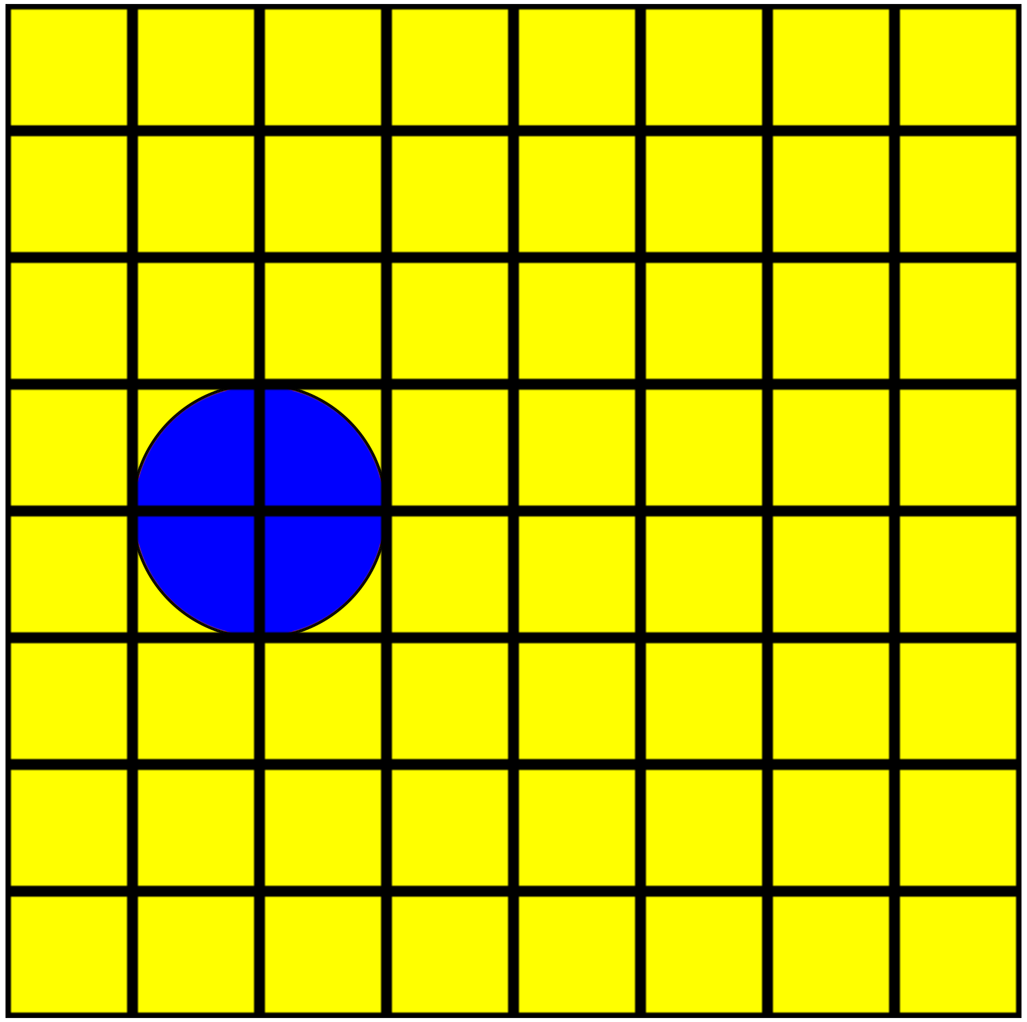


4×4 array of 256×256 grids

This will improve cache performance enormously

Subdivide the domain

3813.7s (64min)



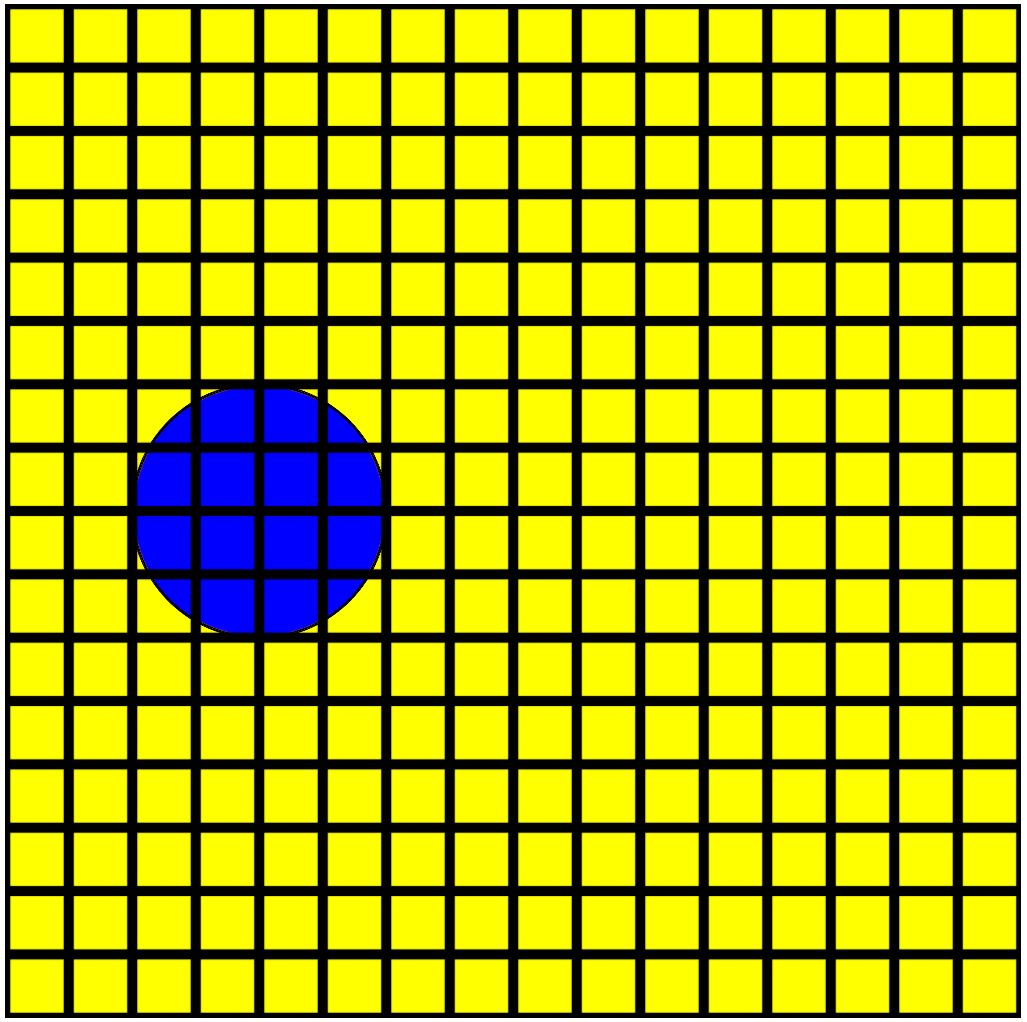
;

8×8 array of 128×128 grids

This will improve cache performance enormously

Subdivide the domain

3865.3s (64min)



;

16×16 array of 64×64 grids

This will improve cache performance enormously

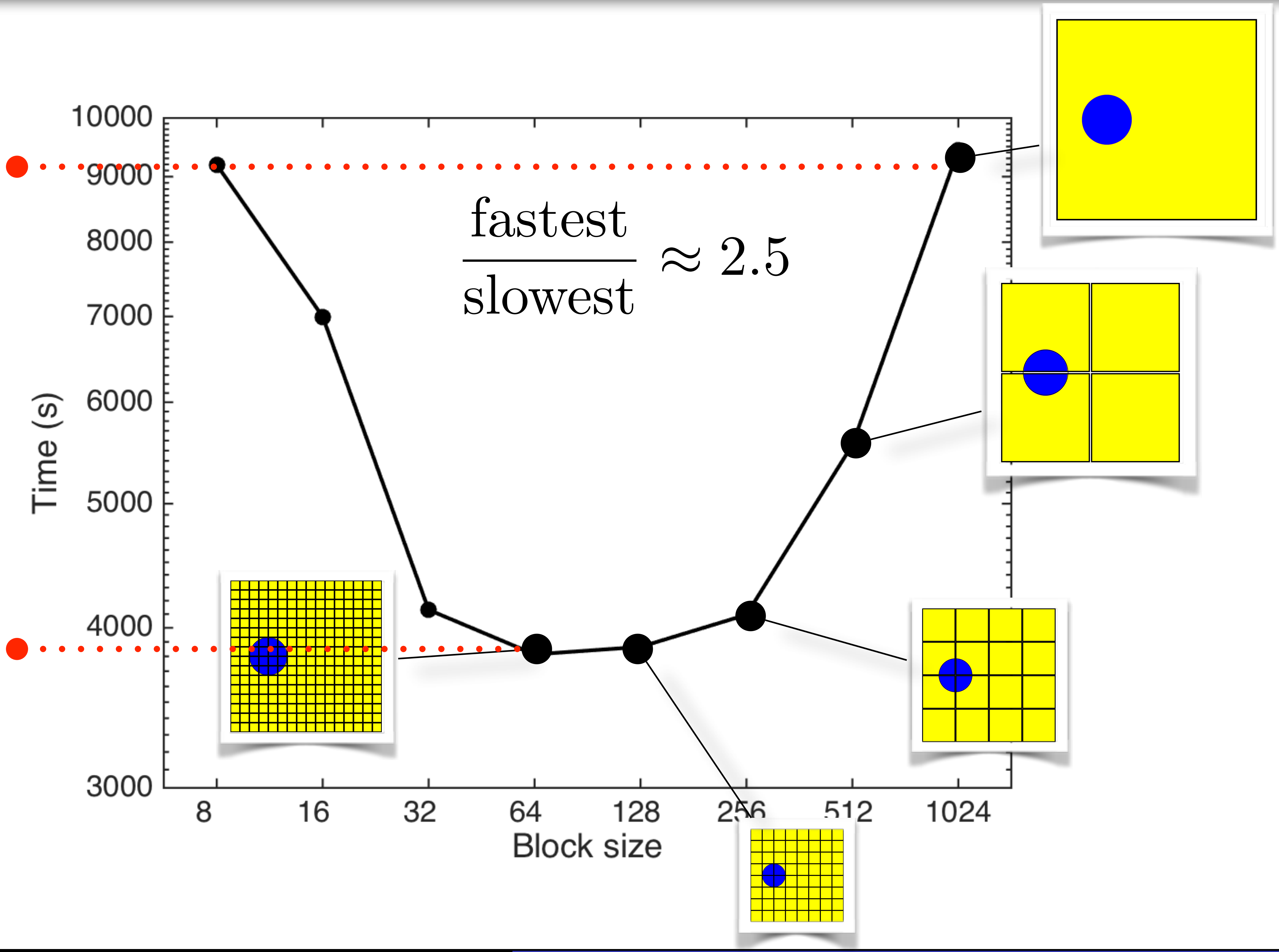
Timing vs block size

9194.5s (1)

(2.5 hrs)

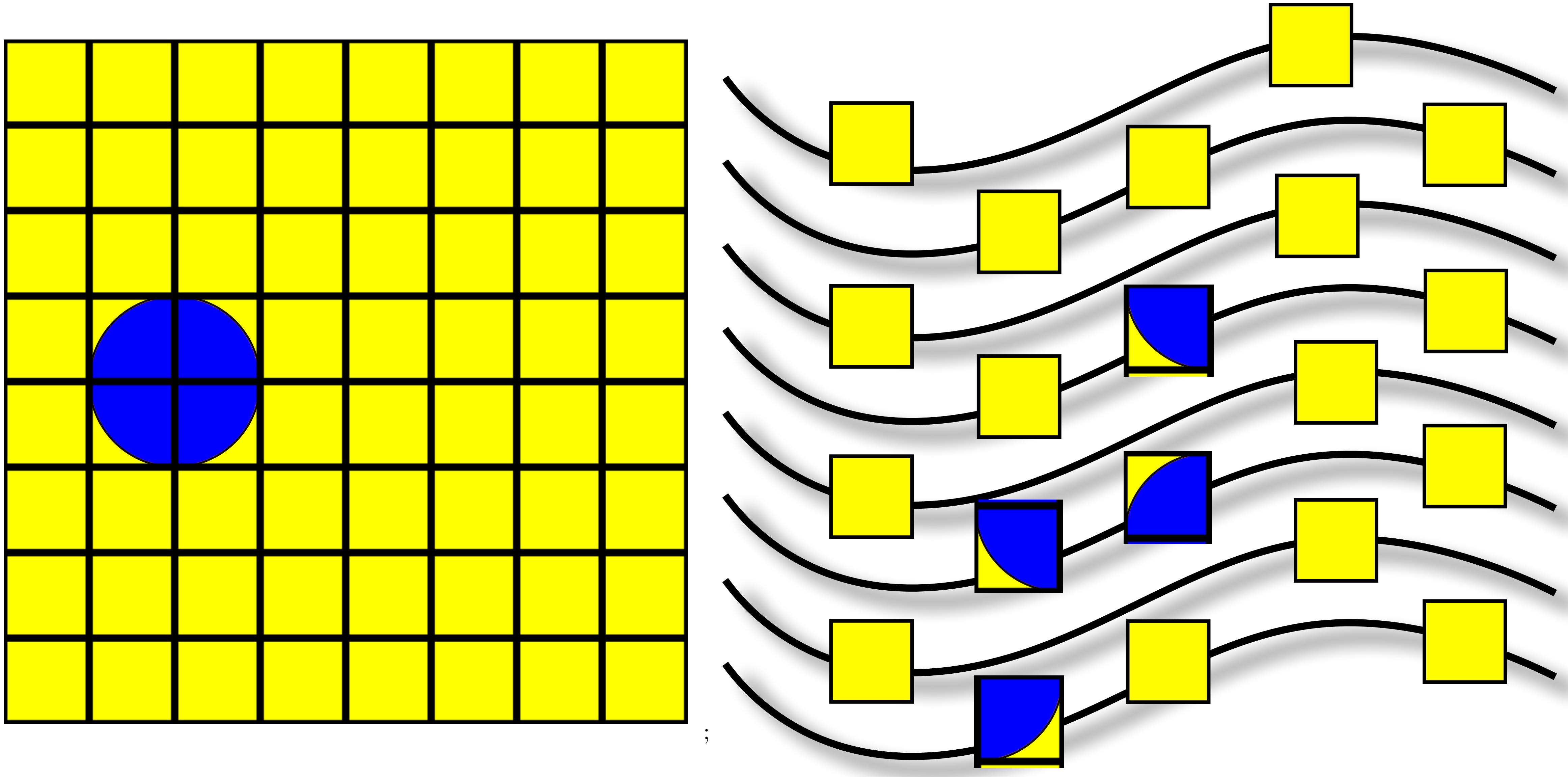
3813.7s (2.4)

(1.1 hrs)



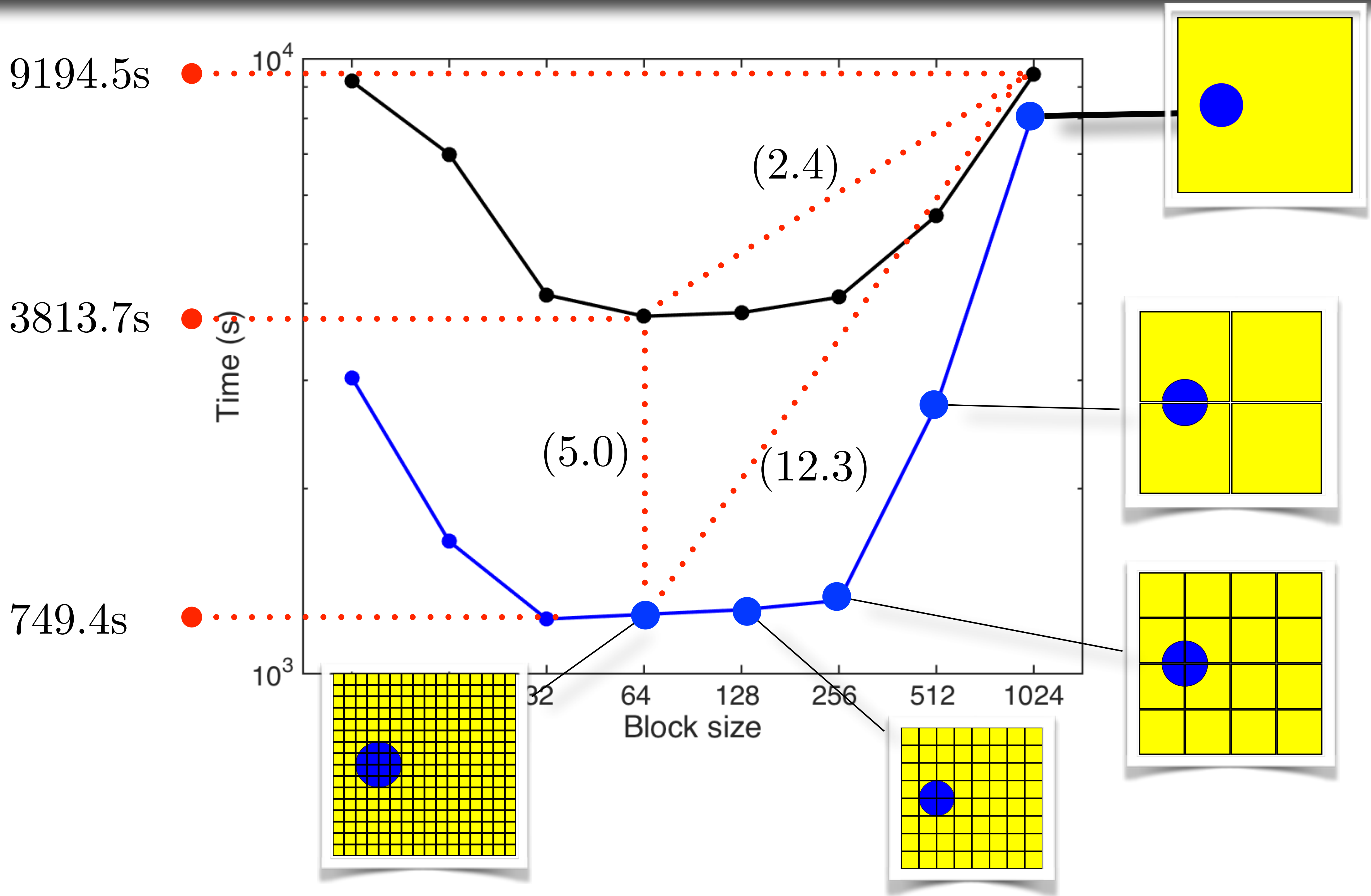
$$\frac{\text{fastest}}{\text{slowest}} \approx 2.5$$

Use multiple cores



Distribute patches to hardware threads. On a four core machine, this should lead to about a factor four speedup.

Speed up using 4 cores (Intel i7)



Can we improve on this?

Hardware solutions : Use more computing units (cpus/threads/cores)

-- Eventually, however, the communications costs will overwhelm any gains made in subdividing the domain further

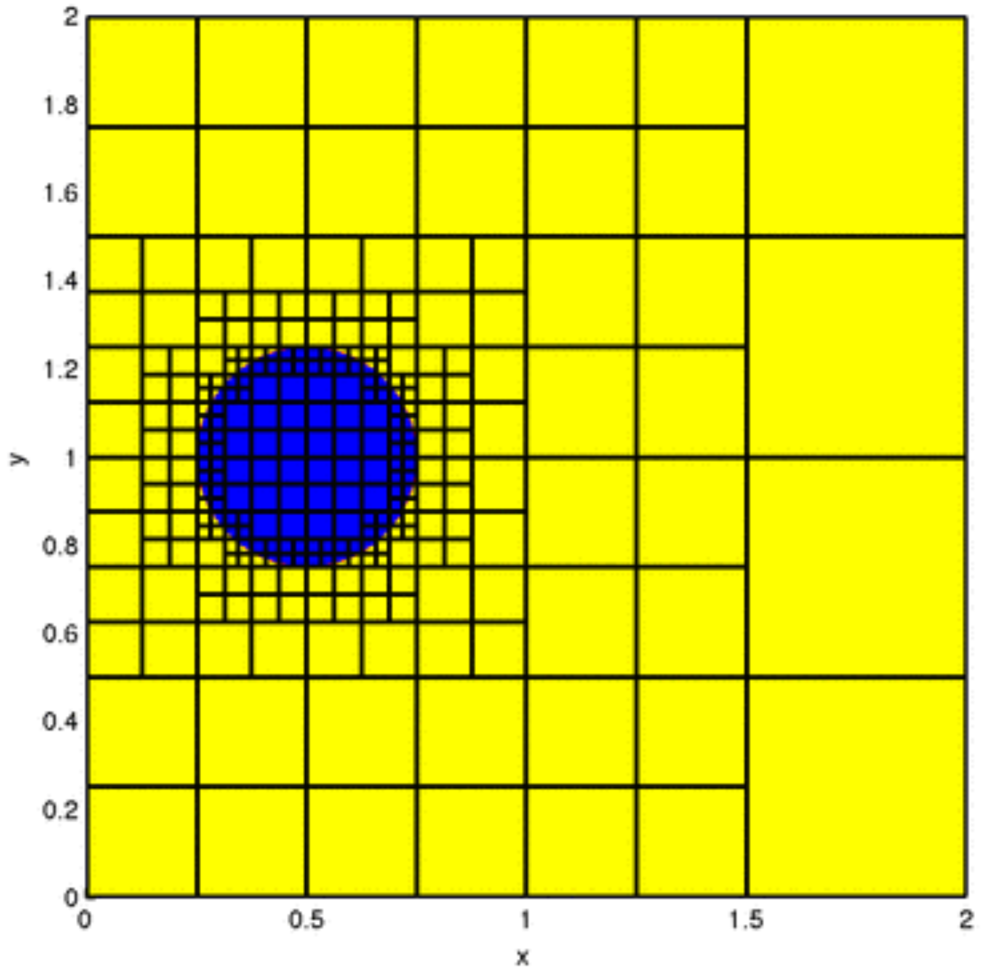
-- Problems always outgrow the hardware that is available

Software solutions : Reduce the "factor 8" scaling law and put resources only where they are needed.

-- This is much more difficult than just adding more computing units

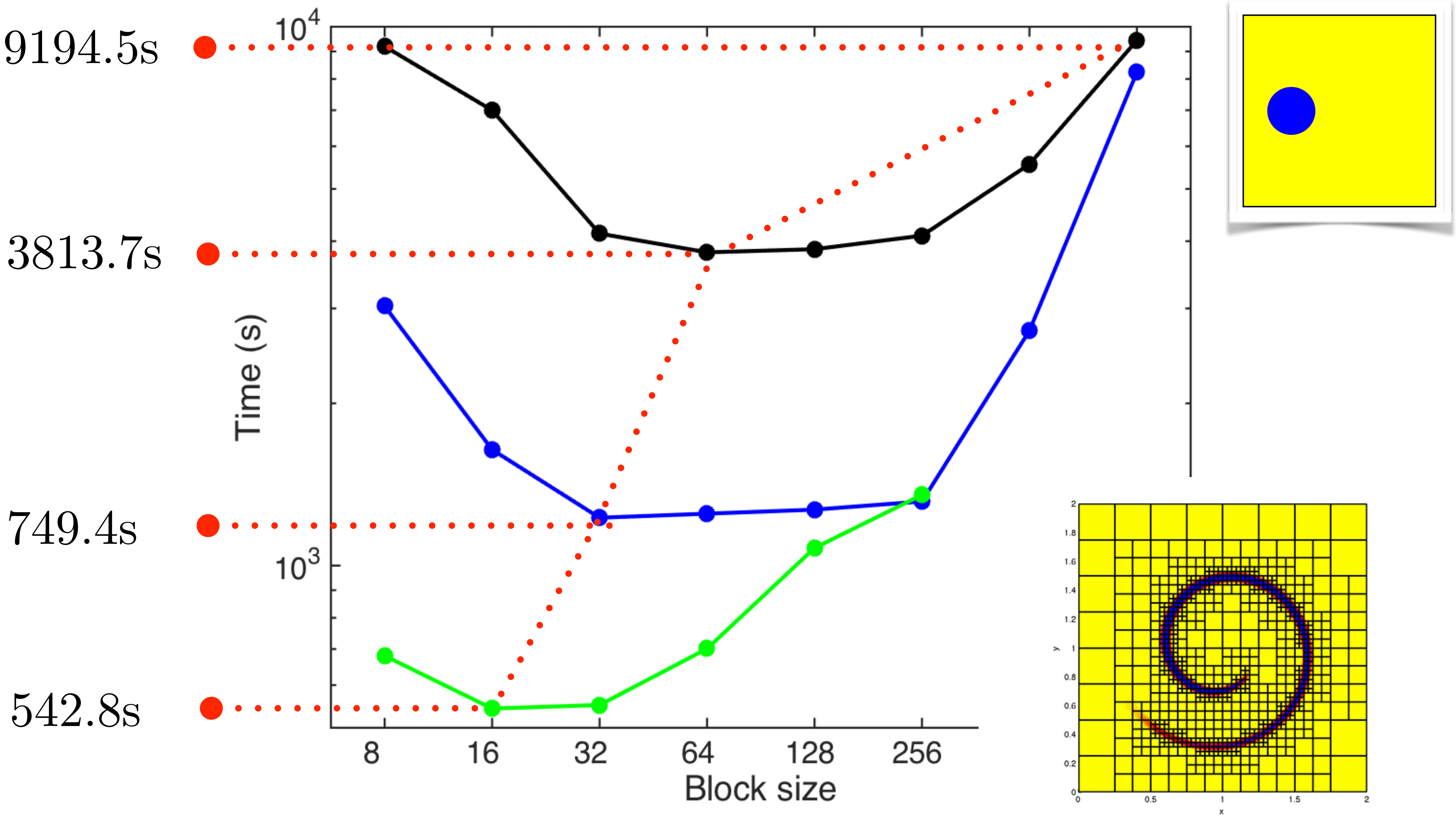
-- Should complement hardware advances

AMR simulation



Effective 512 x 512 resolution (116s vs. 917s)

With adaptive mesh refinement



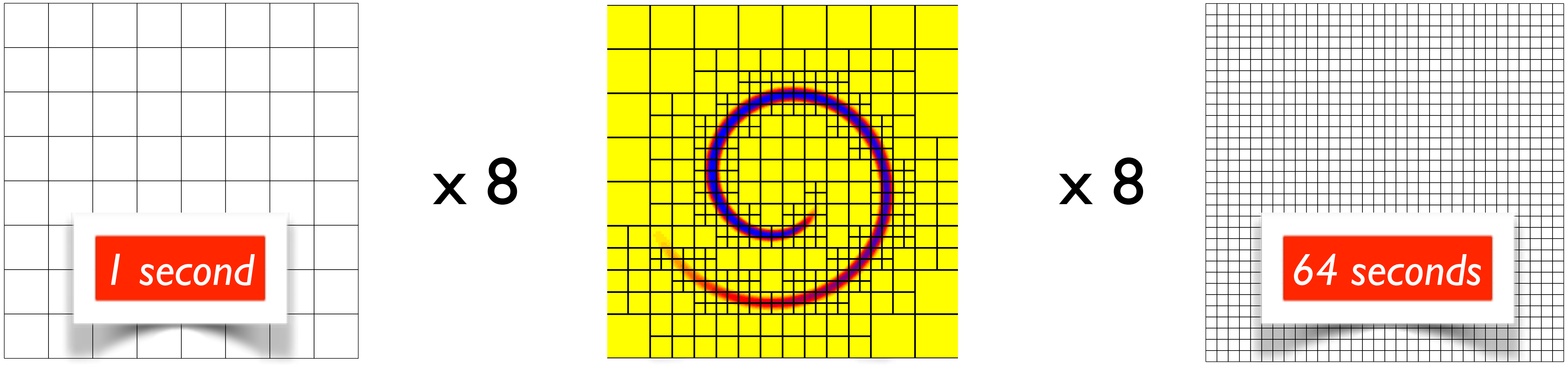
Almost a factor of 15 improvement !

One more tweak ...

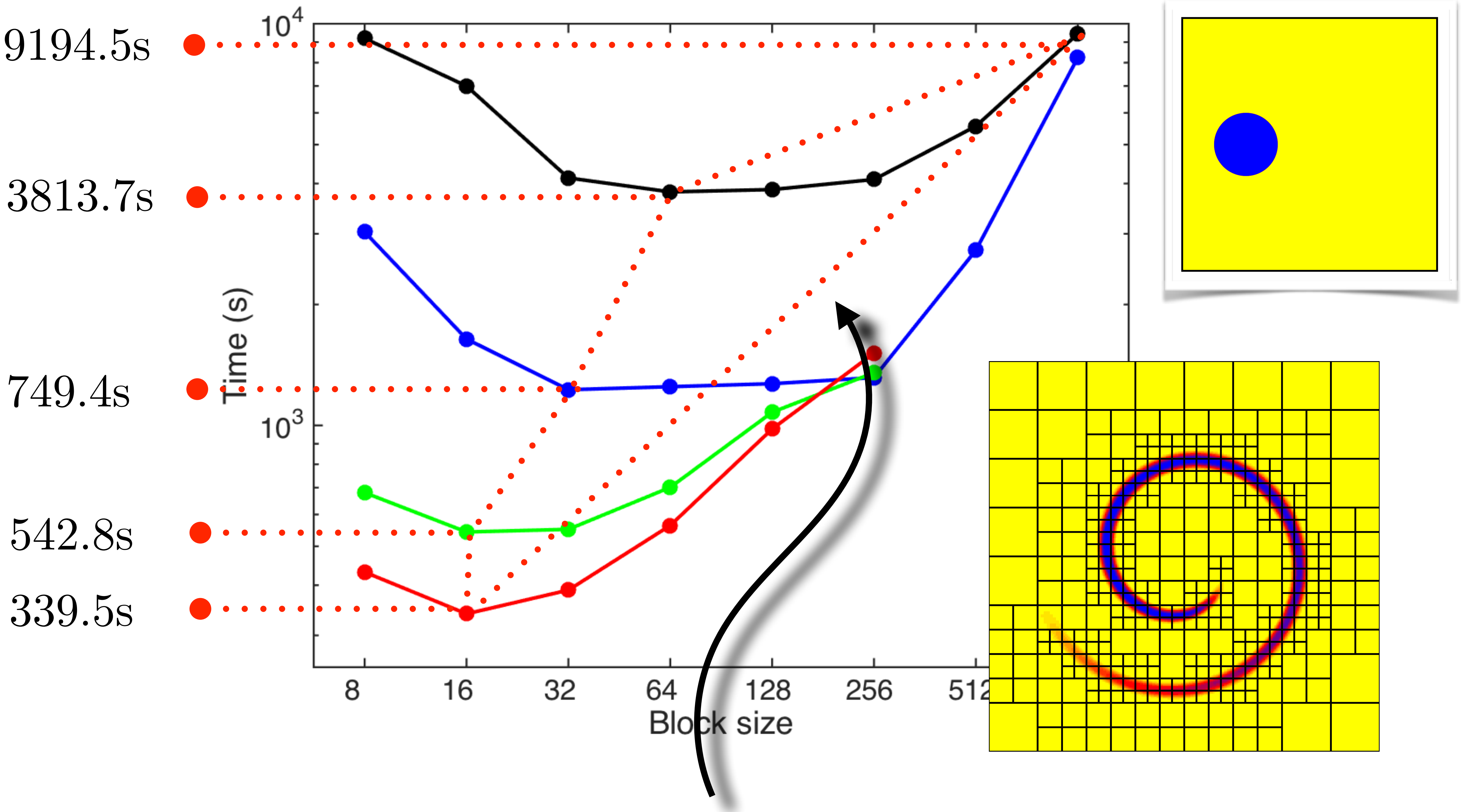
In two dimensions, if we double the resolution, we increase the computational cost by a factor of 8

$$\begin{gathered}\text{(Number of grid cells increases by a factor of 4)}\times\\ \text{(Number of time steps increase by a factor 2)} =\\ \text{(8-fold increase in computational expense)}\end{gathered}$$

The CFL condition constrains the size time step we can take on a grid. For stability, smaller grid cells need smaller time steps.



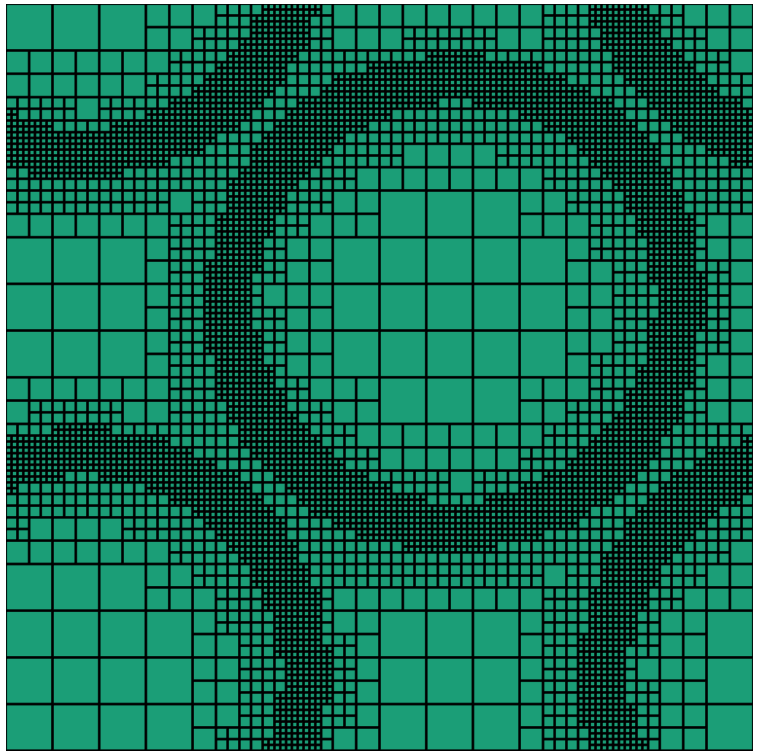
Multi-rate time stepping



Almost a factor of 30 improvement !

Scaling of single adaptive problem

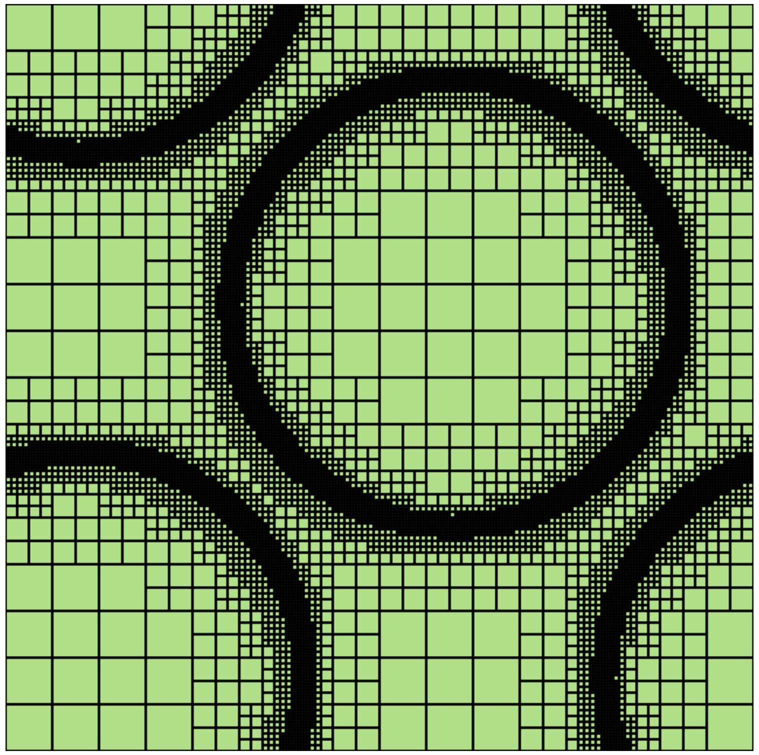
Levels 4-7



level 4 grids

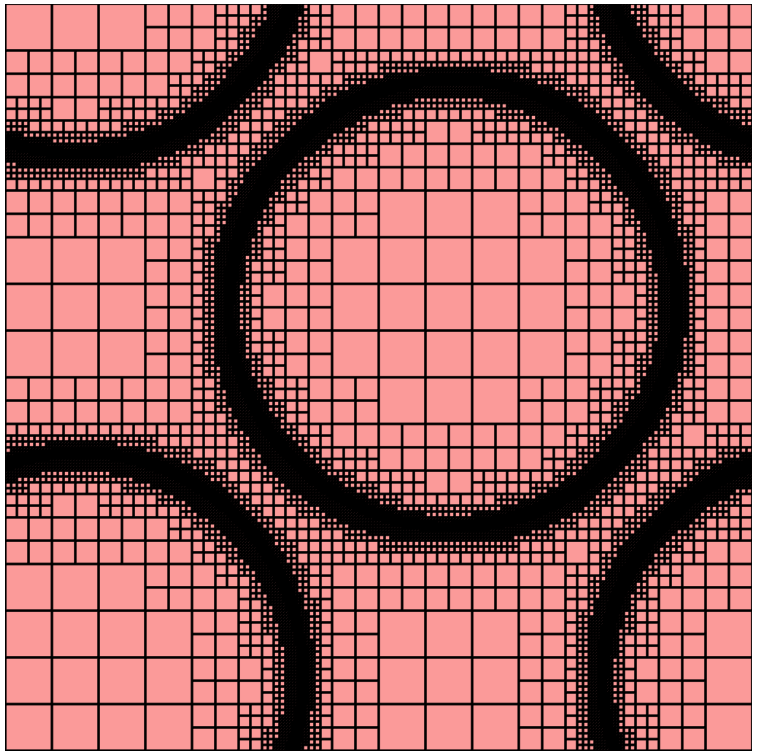
level 7 grids

Levels 4-8



level 8 grids

Levels 4-9



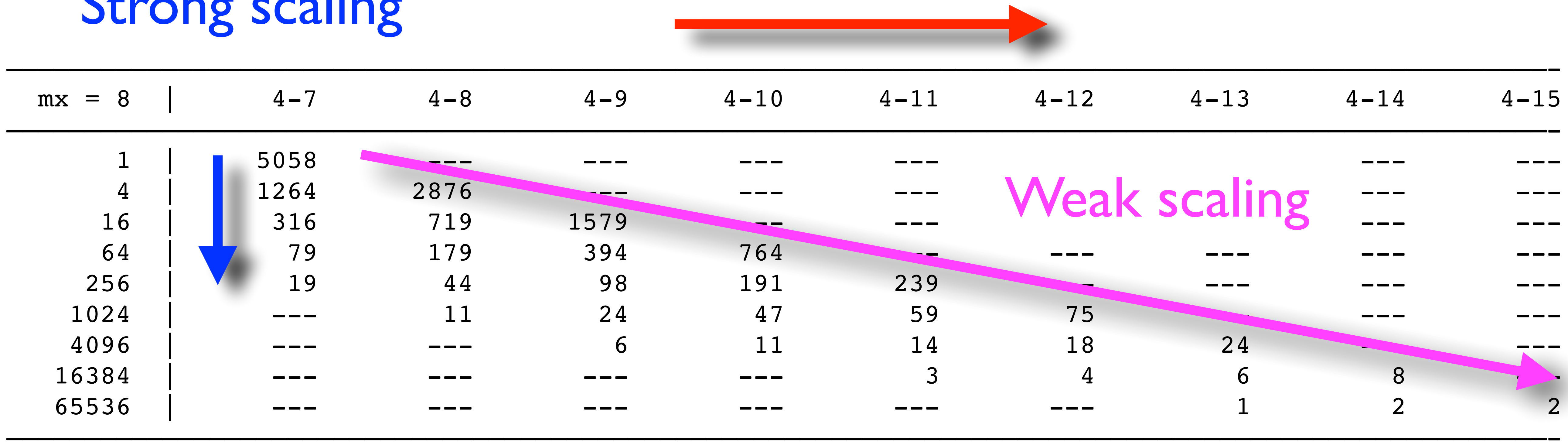
level 9 grids

Mimic scaling of uniform problem : Increase resolution and processor count simultaneously.

Adaptive scaling

Keep minimum level fixed;
Increase maximum refinement level

Strong scaling



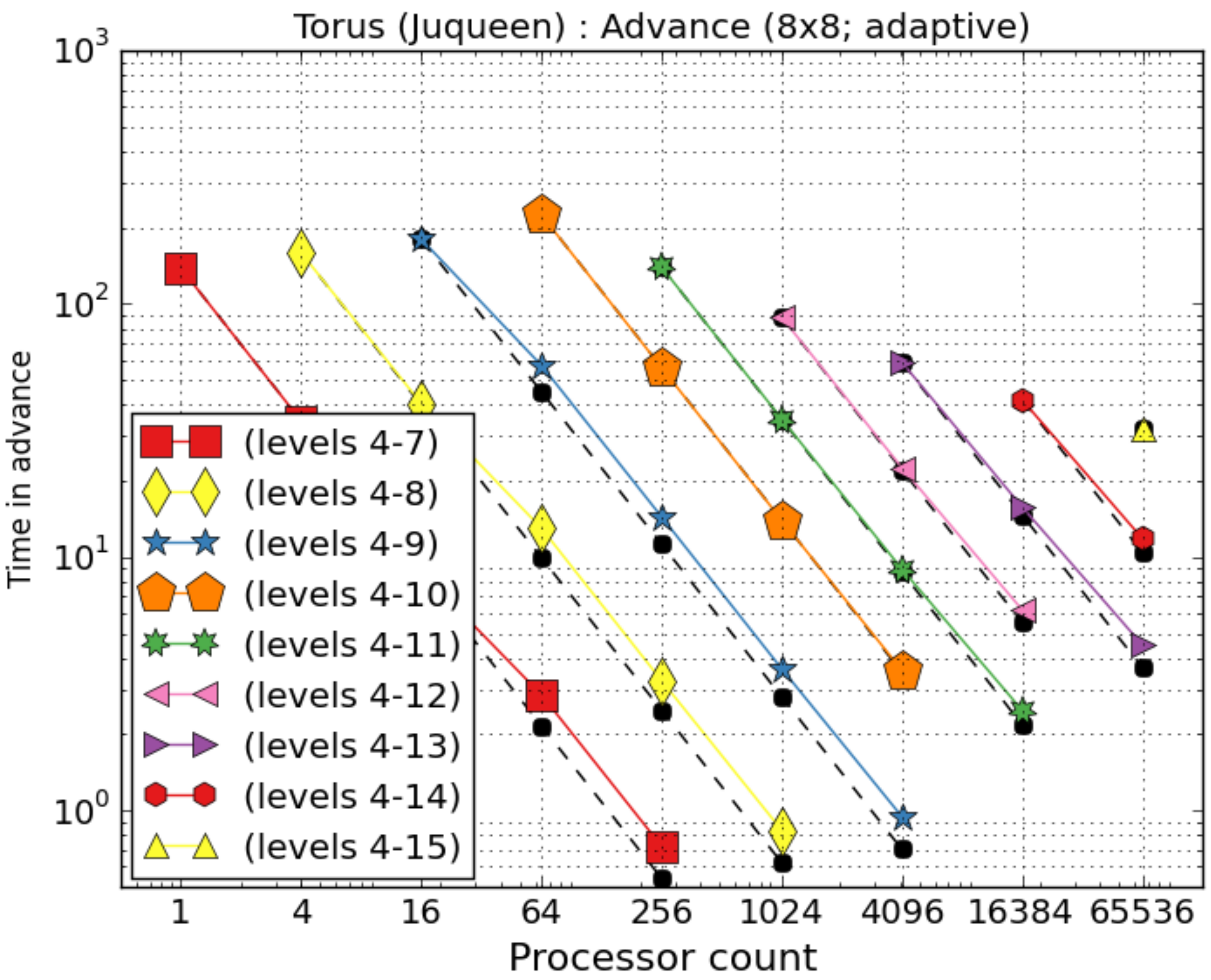
Weak scaling

mx = 8	4-7	4-8	4-9	4-10	4-11	4-12	4-13	4-14	4-15
1	5058	---	---	---	---			---	---
4	1264	2876	---	---	---			---	---
16	316	719	1579	---	---			---	---
64	79	179	394	764	---	---	---	---	---
256	19	44	98	191	239	---	---	---	---
1024	---	11	24	47	59	75	---	---	---
4096	---	---	6	11	14	18	24	---	---
16384	---	---	---	---	3	4	6	8	[illegible]
65536	---	---	---	---	---	---	1	2	2

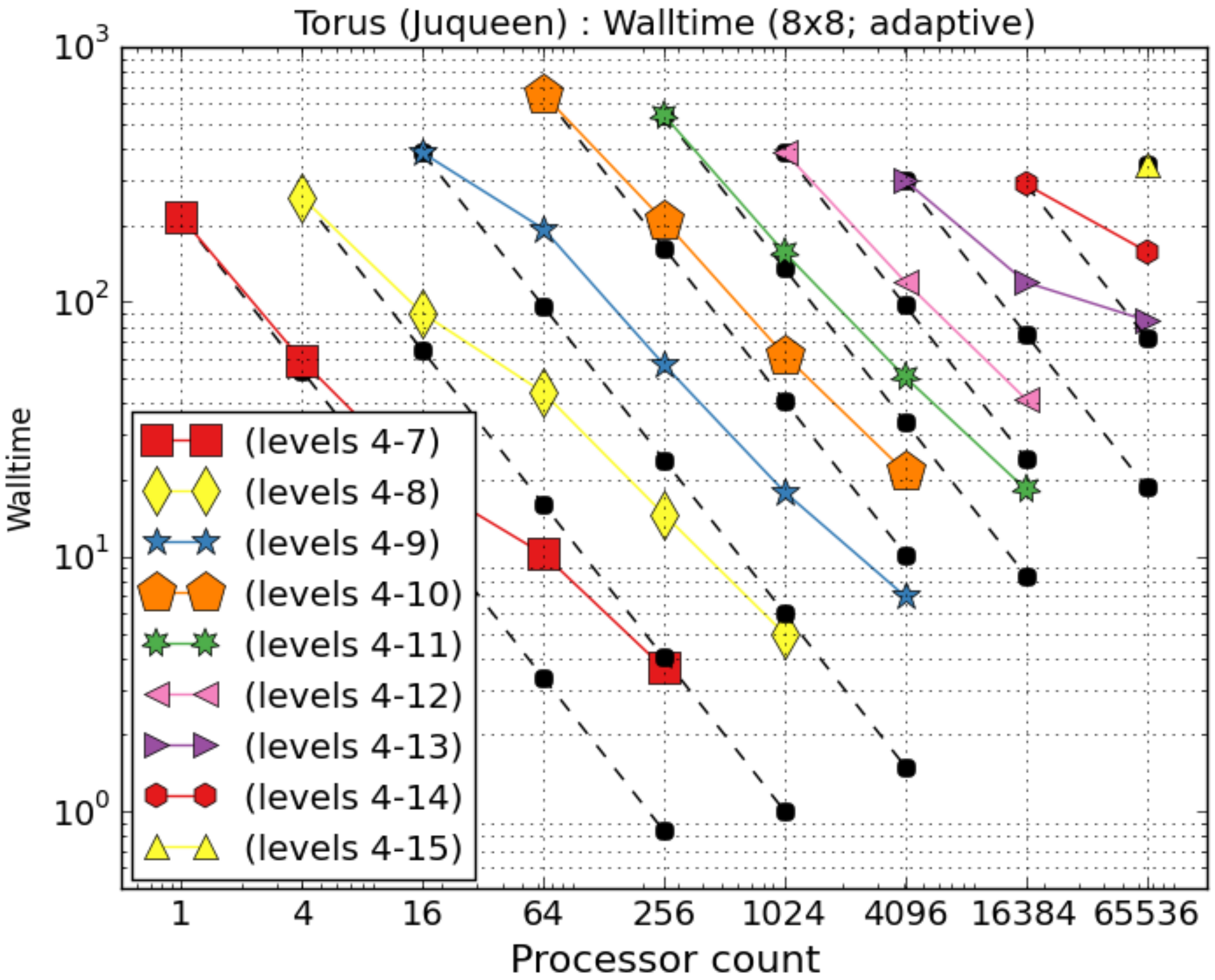
- Fixed number of coarse grid time steps; final time depends on grid size.
- Strong scaling results similar to those for replicated problem.

Strong scaling - adaptive - 8x8

Time in advance



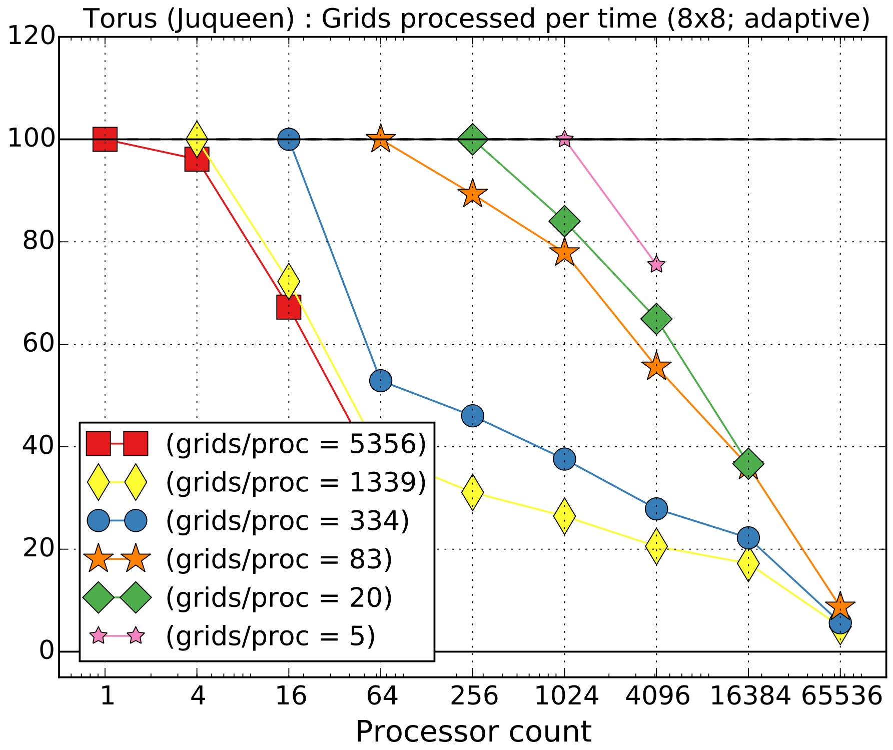
Total walltime



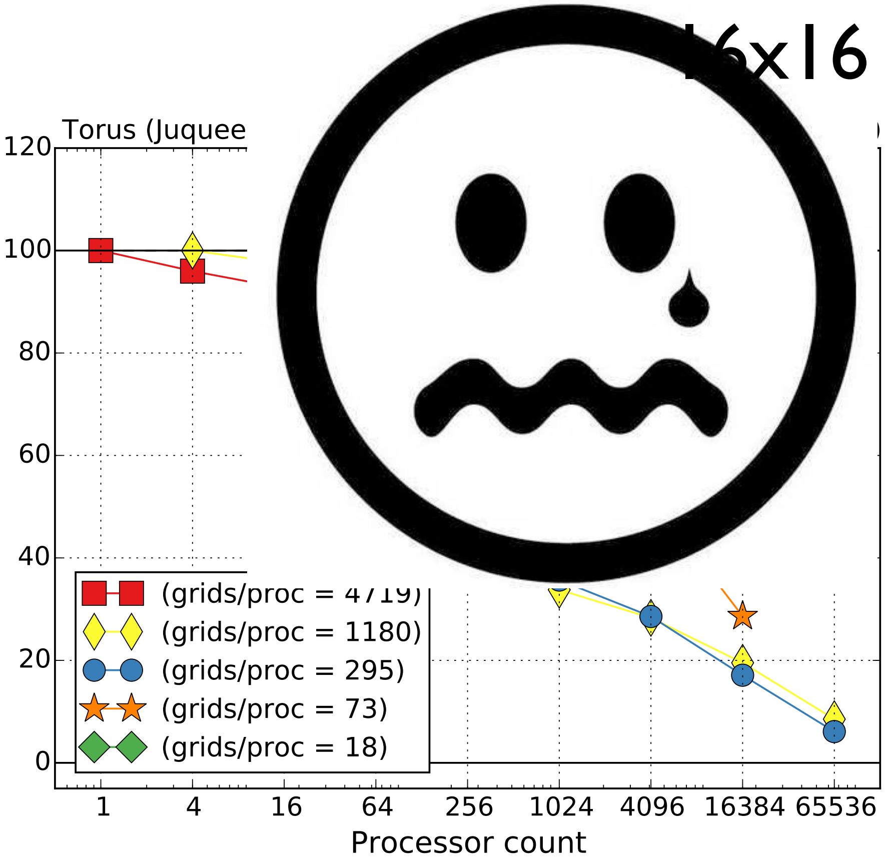
Weak scaling - adaptive

Adaptive (non-replicated) results

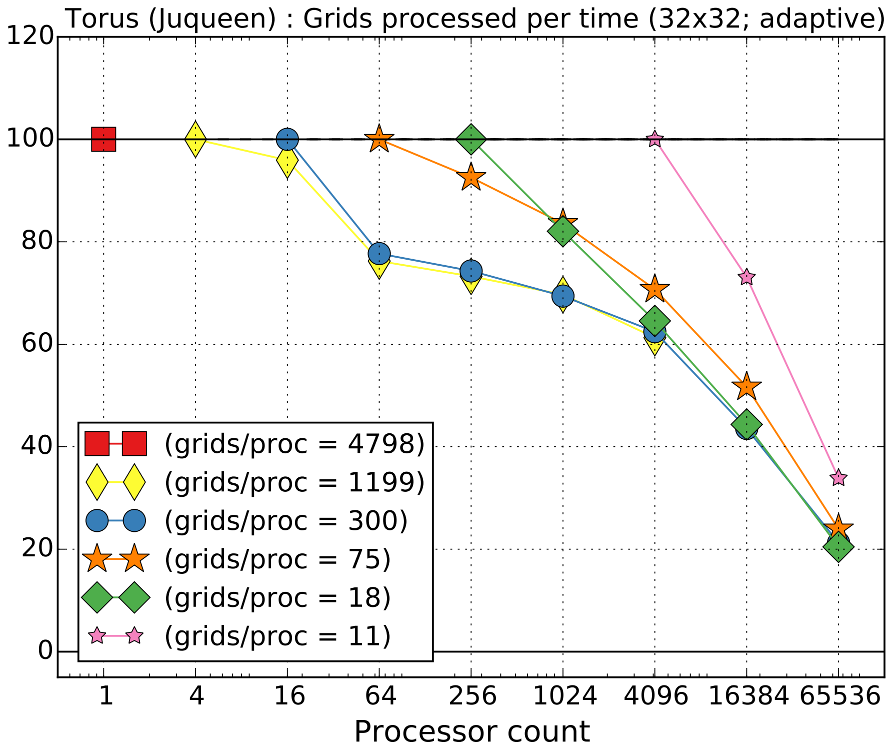
8x8

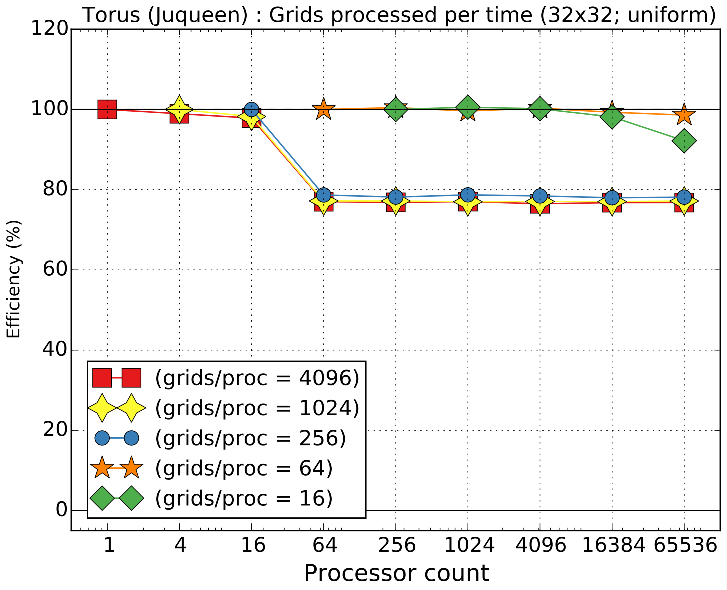


16x16



32x32



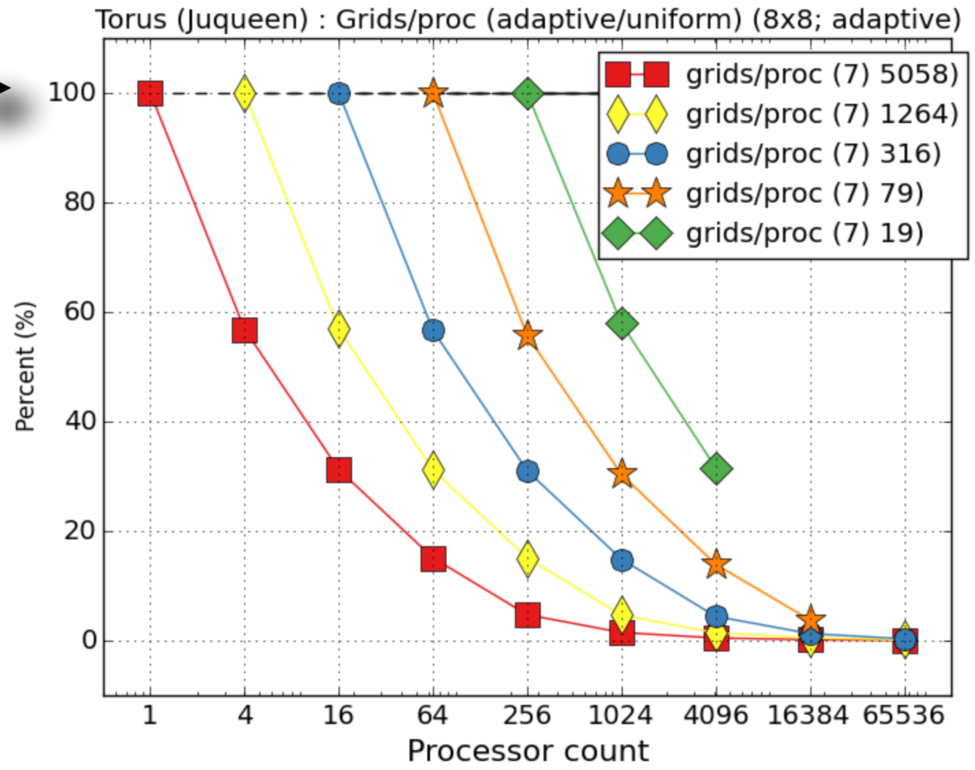


Uniform results

Weak scaling results

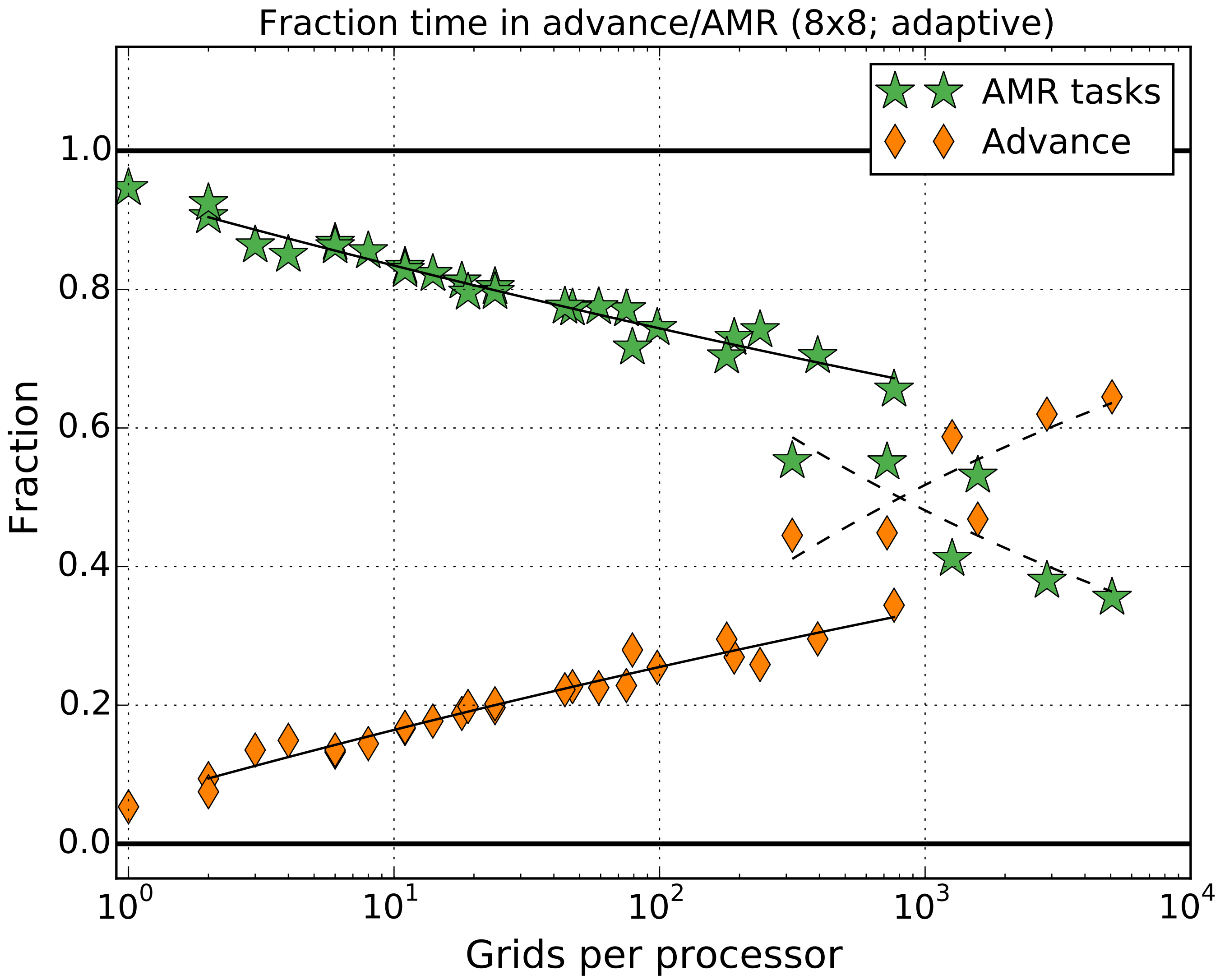
mx = 8	4-7	4-8	4-9	4-10	4-11	4-12	4-13	4-14	4-15
1	**5058**	---	---	---	---	---	---	---	---
4	1264	**2876**	---	---	---	---	---	---	---
16	316	719	**1579**	---	---	---	---	---	---
64	79	179	394	**764**	---	---	---	---	---
256	19	44	98	191	**239**	---	---	---	---
1024	---	11	24	47	59	**75**	---	---	---
4096	---	---	6	11	14	18	**24**	---	---
16384	---	---	---	---	3	4	6	**8**	---
65536	---	---	---	---	---	---	1	2	**2**

Uniform/replicated scaling



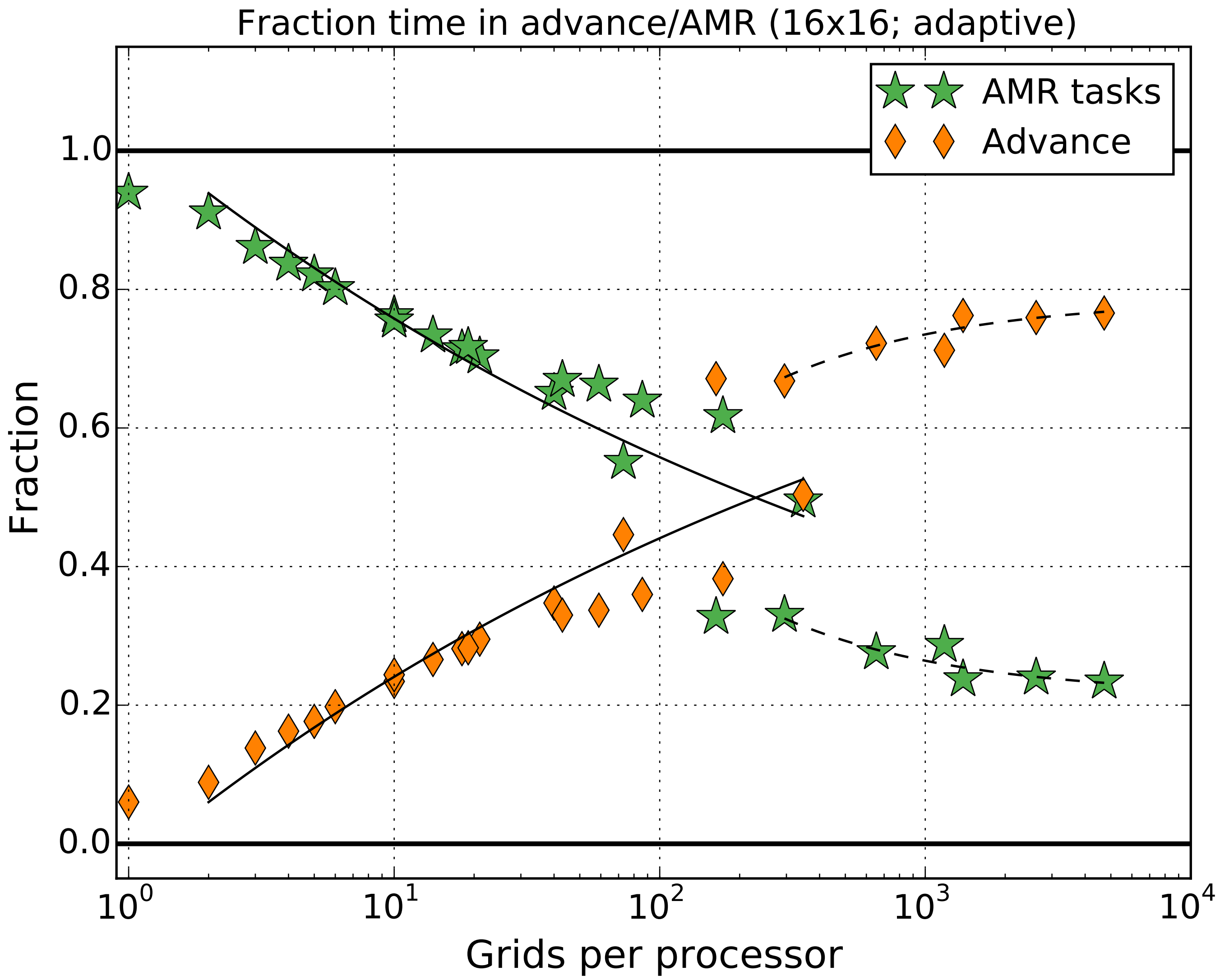
Grids per processor, as percent of grids on processor on original problem.

Adaptive scaling - 8x8



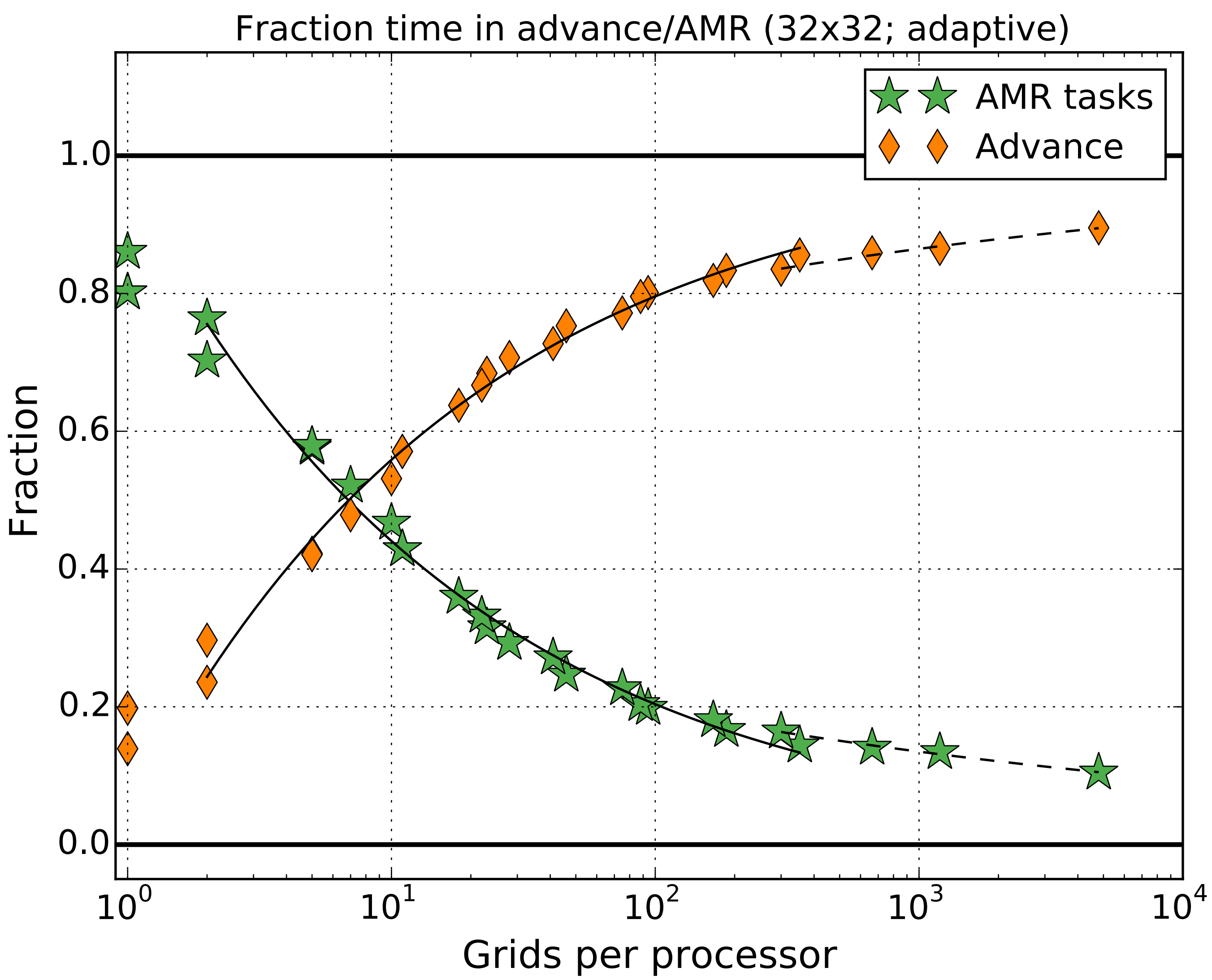
Plot timing results as function of granularity

Adaptive scaling - 16x16



Plot timing results as function of granularity

Adaptive scaling - 32x32



Plot timing results as function of granularity

Backup Slides

Is AMR really worth it?

Skepticism about AMR

Still, there remains skepticism about how effective adaptive mesh refinement (AMR) can be

- Coarse/fine boundaries with abrupt resolution changes are regarded with suspicion,
- Lack of good refinement criteria dampens enthusiasm for trying out AMR,
- Not obvious how to extend sophisticated numerical algorithms and applications to the adaptive setting,

When AMR *is* used,

- Multi-rate time stepping is not always used,
- The goals are often modest :"Do no harm!"
- Grids are often only static; not dynamically refined.

Why are AMR codes difficult to write and use?

- Heterogeneous data structures associated with storing hierarchy of grids,
- Dynamically creating and destroying grids :
- Need a "factory" paradigm to create new grids and any auxiliary data arrays (material properties, metric terms, bathymetry, etc) that go with each new grid,
- Parallel load balancing
- Coupling of multiple solvers (e.g. advection + diffusion + elliptic solvers + source terms),
- Multi-physics,
- Data visualization and post-processing
- Computing diagnostics on a nested grid hierarchy,
- Error estimation, tuning for efficient use of grids, ...

What makes AMR codes difficult to use?

- Time stepping beyond just single step explicit schemes (IMEX, SSP, RK, ...)
- Understanding how overall time stepping interacts with dynamic grid creation, destruction and management.
- Implicit solvers,
- Coupling of multiple solvers (e.g. advection + diffusion + elliptic solvers + source terms),
- Multi-physics,
- Data visualization and post-processing
- Computing diagnostics on a nested grid hierarchy,
- Error estimation, tuning for efficient use of grids, ...

Backup slides

Scientific Software Development

Thanks!

** The Big Bang Theory*

Scientific Software Development …

Backup slides

Existing AMR Codes

What if you have ideas about

- Multi-stage, multi-step, IMEX, SSP, parallel-in-time, exponential integrators, and other time stepping schemes in an adaptive setting,
- Accuracy of multi-rate schemes for PDEs with mixed elliptic/parabolic/hyperbolic terms.
- Elliptic and parabolic solvers (iterative? direct? Explicit? Fast multipole?)
- Parallelism in the AMR setting?
- Error estimation
- Higher order accuracy
- Complex physics

Should you write yet-another-AMR code?

What they might be thinking...

We'll just refine the grid everywhere

My grant doesn't pay for software development

I know somebody who tried it (in 1992) and it didn't work

It year(s) of effort it will take won't pay off

Or...

They've actually tried it...

Block-structured AMR codes

- General purpose (freely available) block-structured codes
  - PARAMESH (NASA/Goddard)
  - SAMRAI (Lawrence Livermore National Lab)
  - BoxLib (Lawrence Berkeley Lab)
  - Chombo (Lawrence Berkeley Lab)
  - AMRClaw (University of Washington/NYU)
- All are large frameworks, with many developers
- Mostly C++ and Fortran libraries (no GUIs) that started life as research codes.

See my website for a list of many more application specific codes

AMR Software

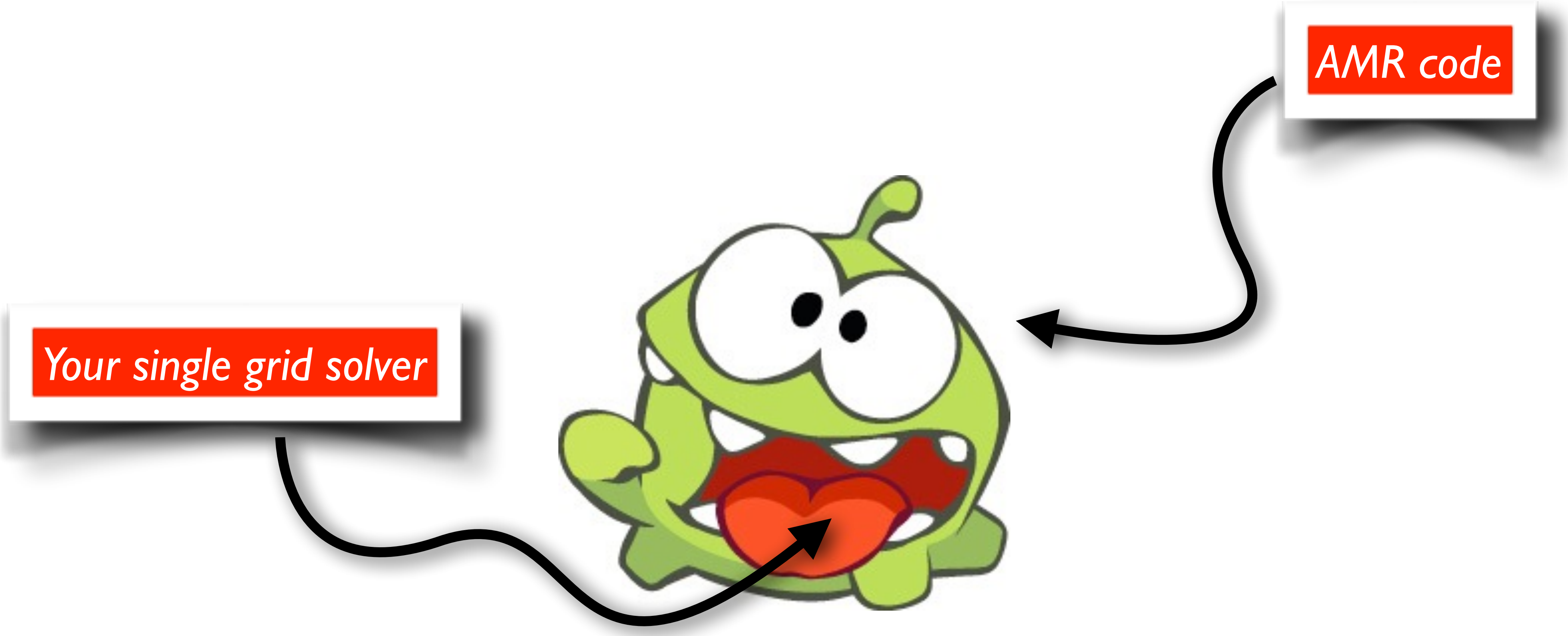
"PARAMESH is a package of Fortran 90 subroutines designed to provide an application developer with an easy route to extend an existing serial code which uses logically Cartesian structured mesh into a parallel code with adaptive mesh refinement"

SAMRAI - "Object oriented C++ library developed to provide algorithmic and software support to large scale multiphysics problems relevant to the US Department of Energy (DOE)"

Boxlib - "These libraries provide the software infrastructure for the computational activities (combustion, astrophysics, porous media flow) at the Center for Computational Sciences and Engineering (CCSE) at Lawrence Berkeley Labs"

Block-structured AMR

Mental model of how this might work :



AMR code

Your single grid solver

* *Idea for code name : OmNum*

Block-structured AMR

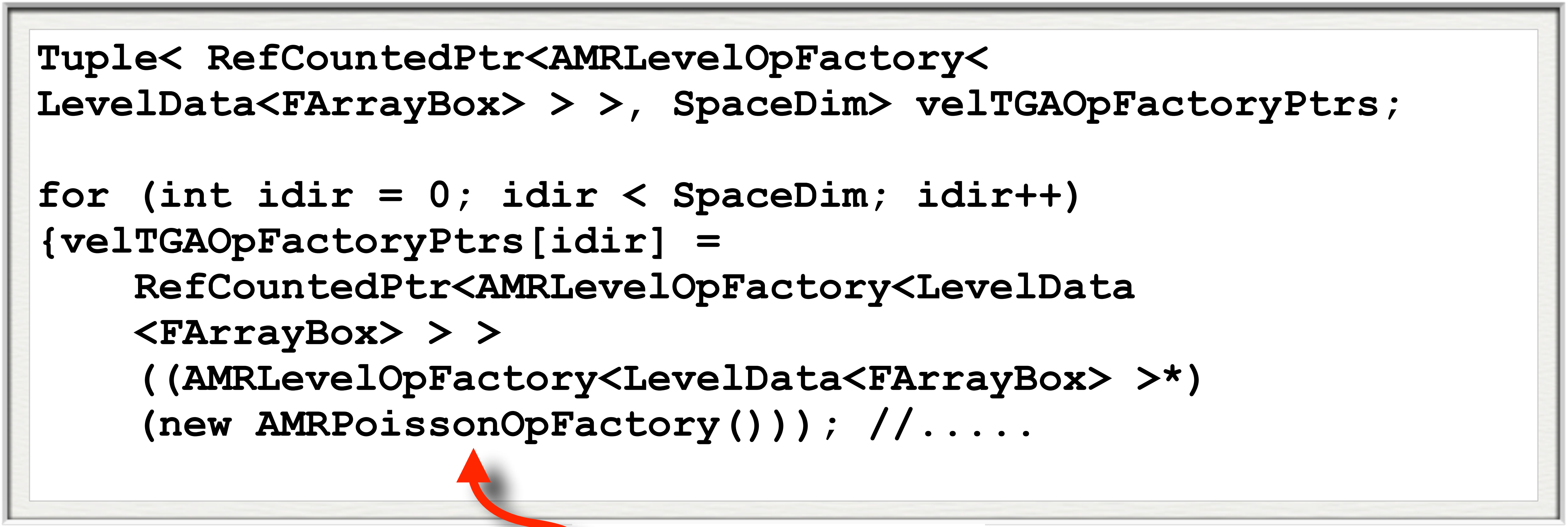
The Dream

```
AMR.run(max_time, max_steps);
```

Your single grid solver is called from here.

Block-structured AMR

The Reality



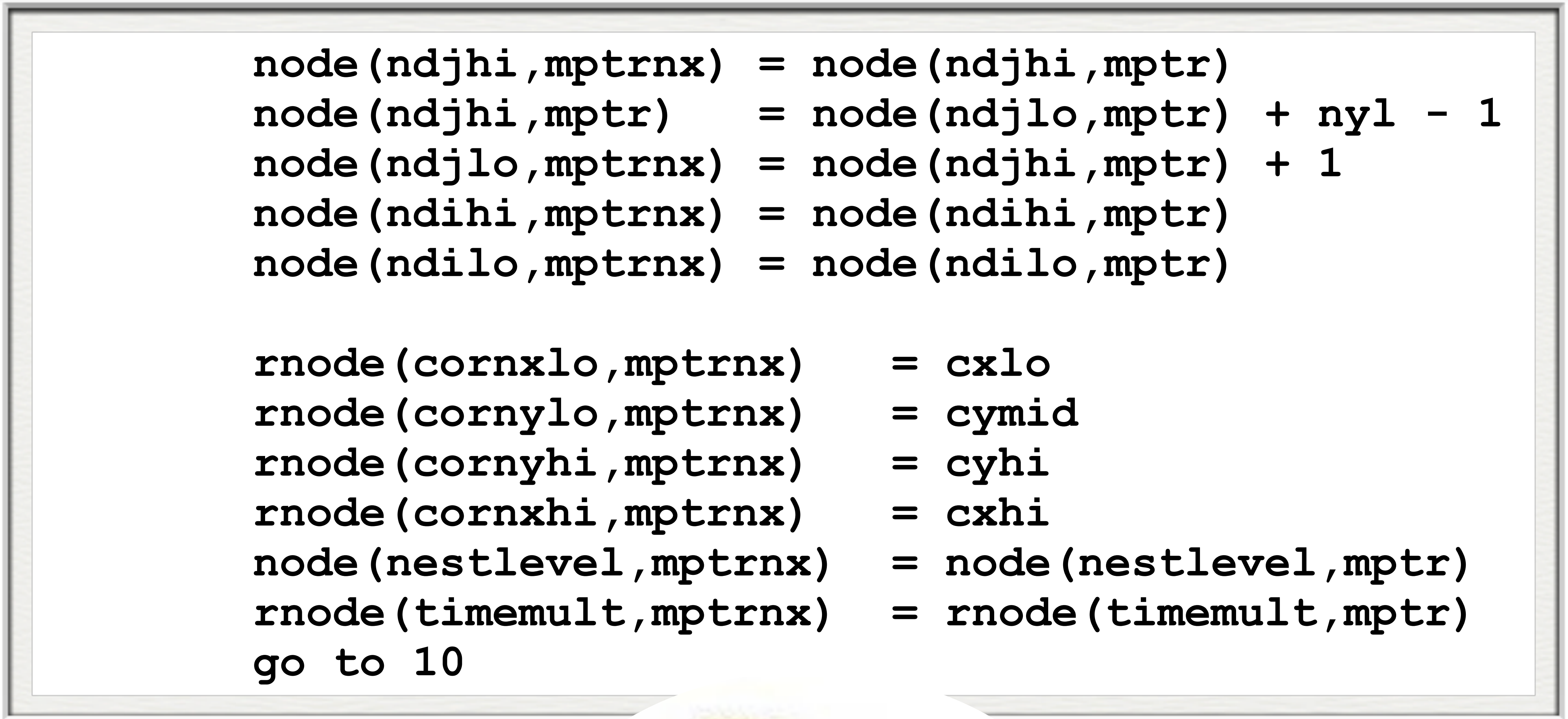
```
Tuple< RefCountedPtr<AMRLevelOpFactory<
LevelData<FArrayBox> > >, SpaceDim> velTGAOpFactoryPtrs;

for (int idir = 0; idir < SpaceDim; idir++)
{velTGAOpFactoryPtrs[idir] =
    RefCountedPtr<AMRLevelOpFactory<LevelData
    <FArrayBox> > >
    ((AMRLevelOpFactory<LevelData<FArrayBox> >*)
    (new AMRPoissonOpFactory())); //.....
```

Your single grid solver

you

Retro...

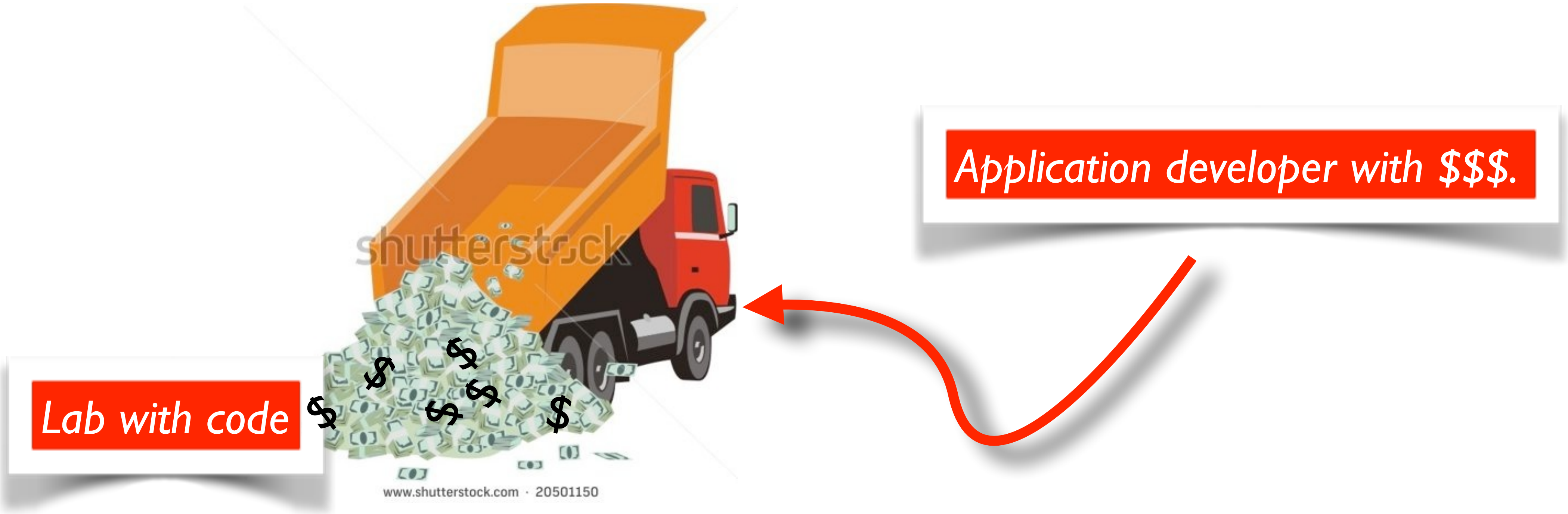


```
node(ndjhi,mptrnx) = node(ndjhi,mptr)
node(ndjhi,mptr)   = node(ndjlo,mptr) + nyl - 1
node(ndjlo,mptrnx) = node(ndjhi,mptr) + 1
node(ndihi,mptrnx) = node(ndihi,mptr)
node(ndilo,mptrnx) = node(ndilo,mptr)

rnode(cornxlo,mptrnx)    = cxlo
rnode(cornylo,mptrnx)    = cymid
rnode(cornyhi,mptrnx)    = cyhi
rnode(cornxhi,mptrnx)    = cxhi
node(nestlevel,mptrnx)   = node(nestlevel,mptr)
rnode(timemult,mptrnx)   = rnode(timemult,mptr)
go to 10
```

Block-structured AMR codes

Most of these codes were designed mainly to support large scale applications developers and in-house scientific research.



Lab with code

Application developer with $$$.

Also initially designed largely for hyperbolic PDEs and single step methods.

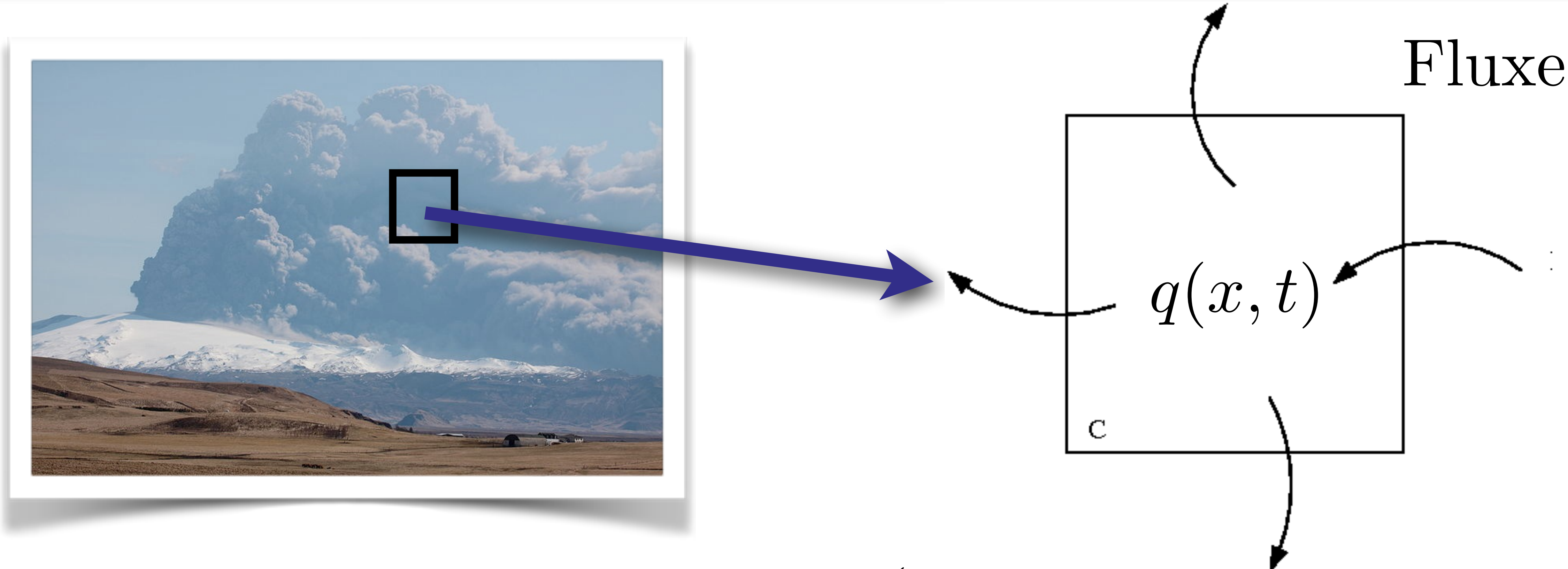
Backup Slides

Natural Hazards Modeling

Backup slides

Finite volume discretization

Modeling physical processes - Finite volumes



A model of the transport of $q(\mathbf{x},t)$ in a control volume C

$$\begin{aligned}\frac{d}{dt}\int_C q(x,t)\,dA &= -\int_C \nabla\cdot\mathbf{f}(q(x,t))\,dA \\ &= -\int_{\partial C}\mathbf{f}(q(x,t))\cdot\mathbf{n}\,dL\end{aligned}$$

Physical fluxes

Fluxes depend on underlying physical model and quantities being conserved, i. e.

- mass, tracer species (ash, smoke, ...) $f(q) = uq$
- heat $f(q) = -\beta q_x$
- momentum $f(\rho, u, E) = u^2\rho + P(\rho, u, E)$
- energy $f(\rho, u, E) = u(E + P(\rho, u, E))$

By updating quantities using fluxes, we can guarantee numerical conservation of mass, energy, momentum, tracers, etc.

Finite Volume Update

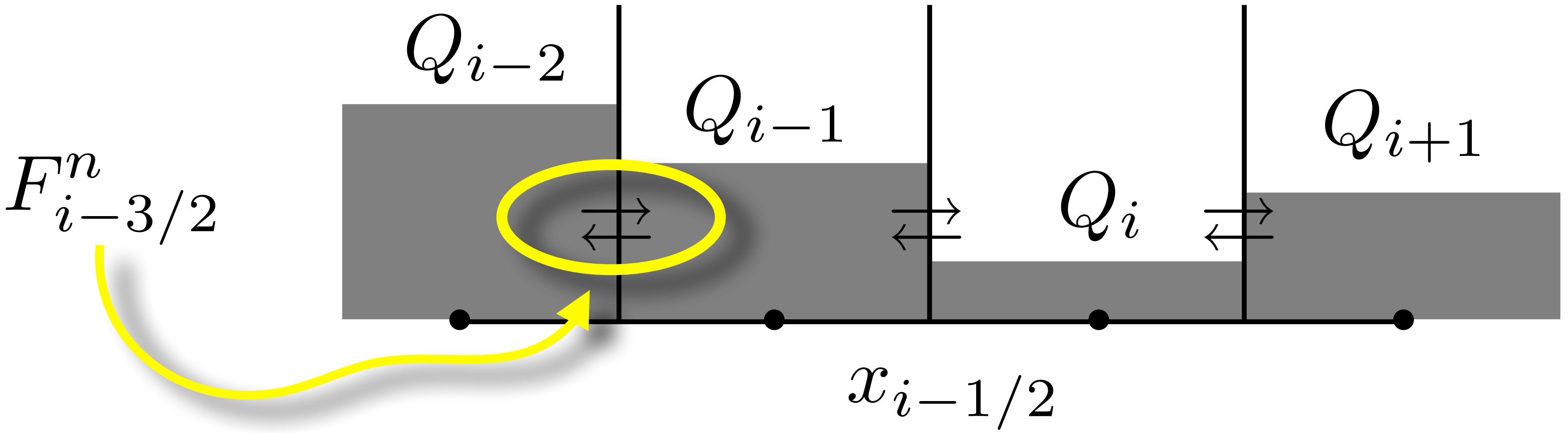
Using numerical fluxes, we use the explicit update formula :

$$Q_i^{n+1} = Q_i^n - \frac{\Delta t}{\Delta x}\left[F_{i+1/2}^n - F_{i-1/2}^n\right]$$

Numerical fluxes

where

$$F_{i-1/2}^n \approx \frac{1}{\Delta t}\int_{t_n}^{t_{n+1}} f(q(x_{i-1/2},t))\,dt$$



Average value in mesh cell

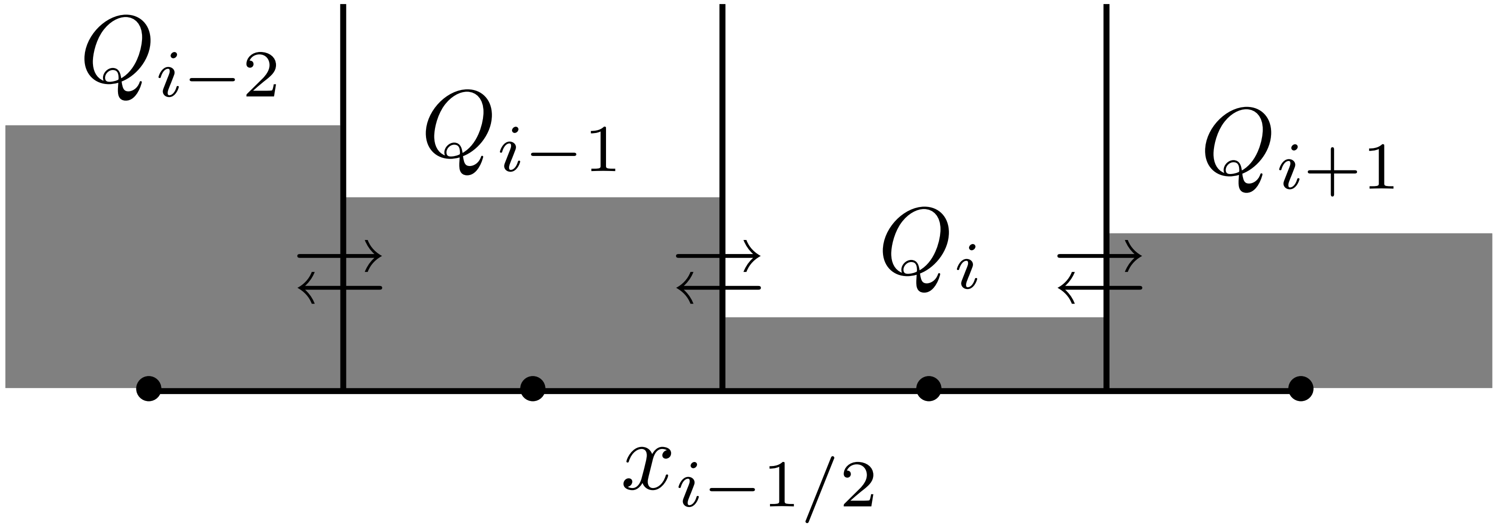
Finite volume schemes

We assume an explicit time stepping scheme

$$F^n_{i-1/2} \approx \int_{t_n}^{t_{n+1}} f(q(x_{i-1/2}, t))\, dt$$

so that we try to find formulas for the flux of the form

$$F^n_{i-1/2} = \mathcal{F}(Q^n_i, Q^n_{i-1})$$



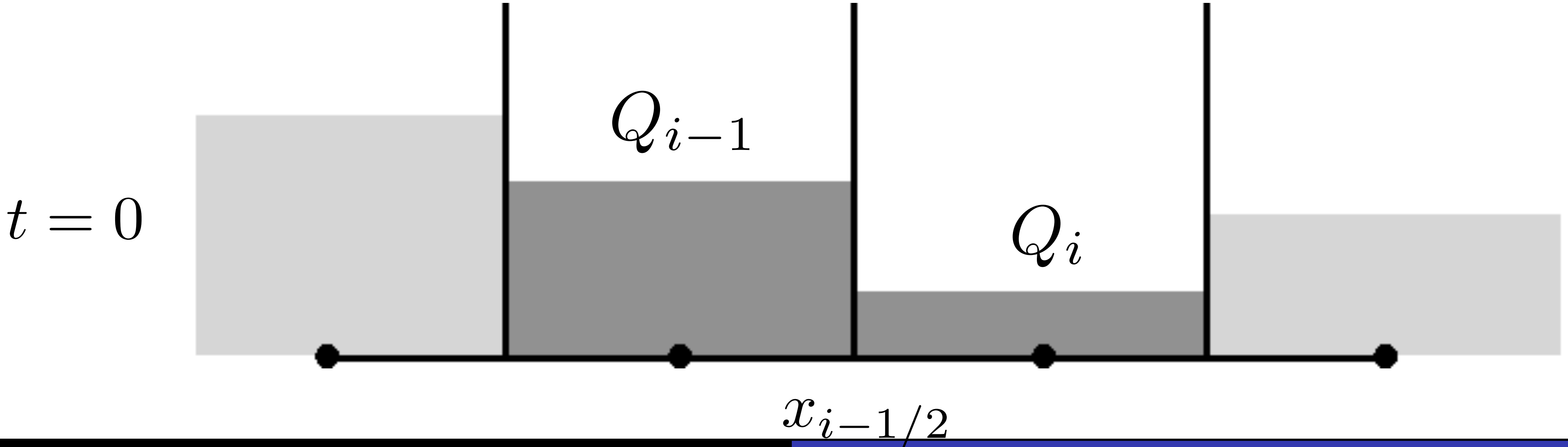
Riemann Problems

At each cell interface, solve the hyperbolic problem with special initial data, i.e.

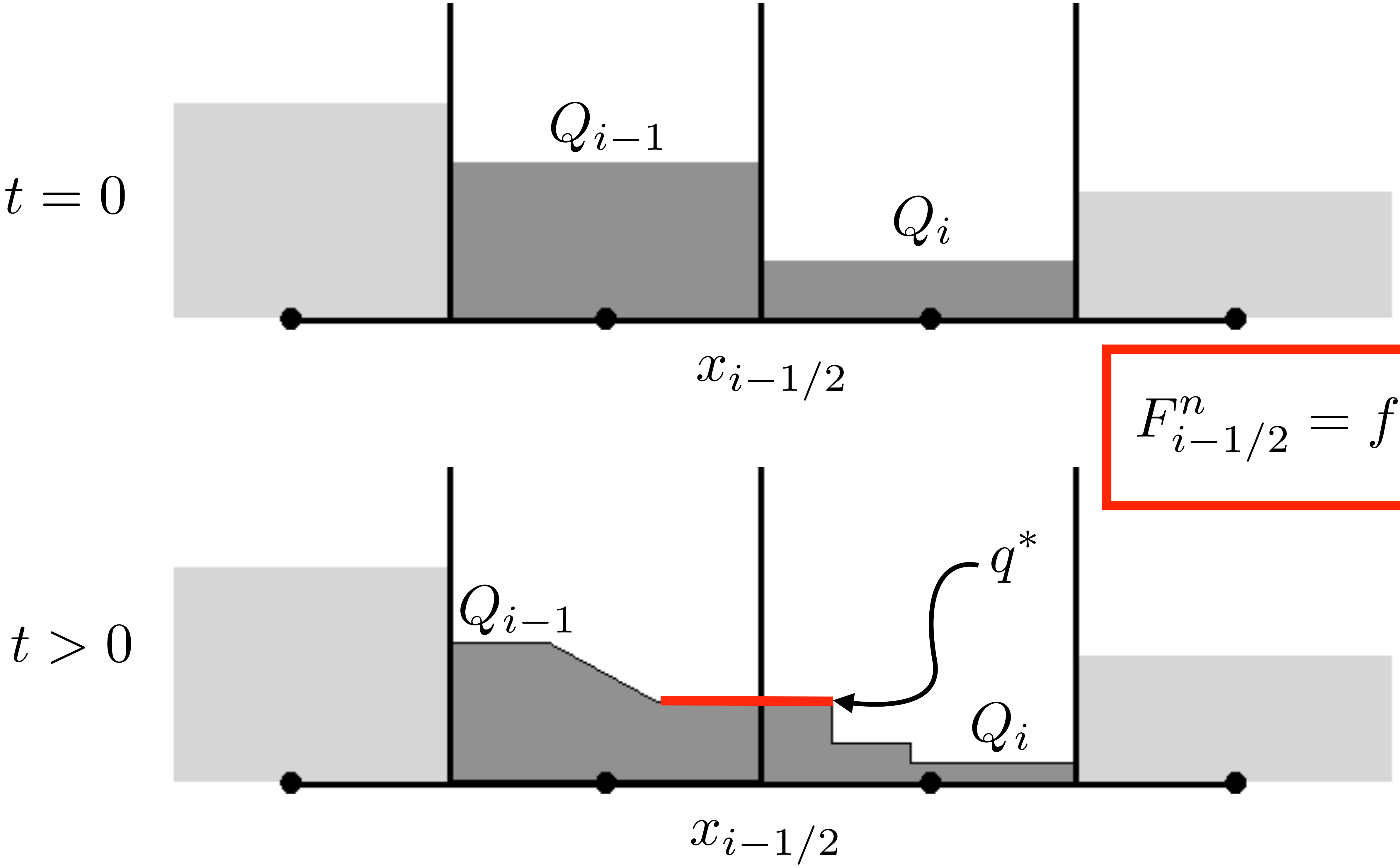
$$q_t + f(q)_x = 0$$

subject to

$$q(x,0) = \begin{cases} Q_{i-1} & x < x_{i-1/2} \\ Q_i & x > x_{x-1/2} \end{cases}$$

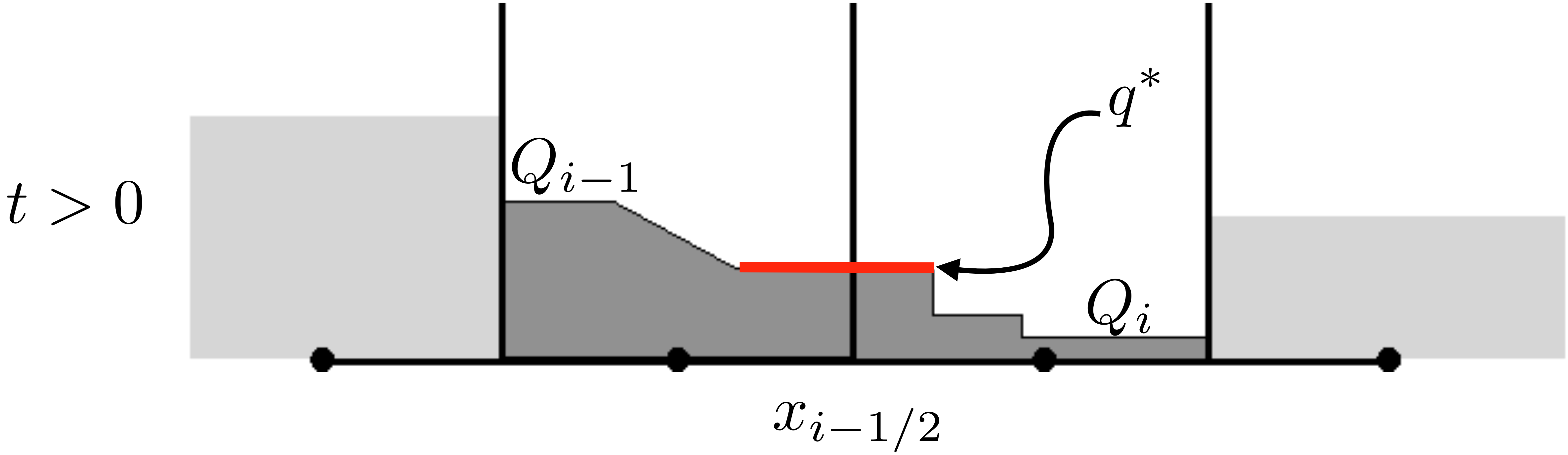


Finite volume update



$$\boxed{F^n_{i-1/2} = f(q^*)}$$

Godunov schemes



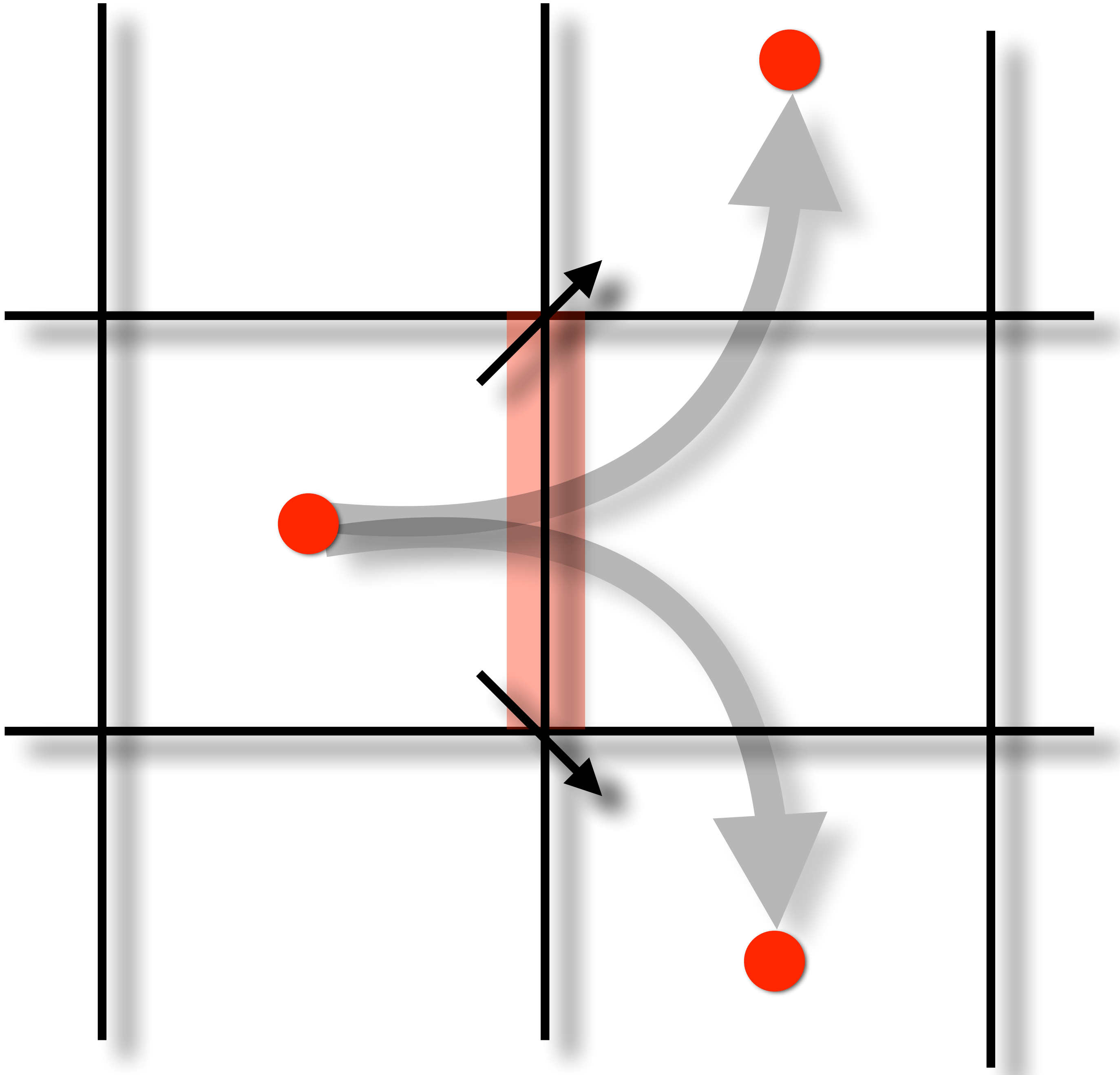
Numerical flux at cell interface is then approximated by

$$F_{i-1/2} = f(q^*)$$

This is the classical Godunov approach for solving hyperbolic conservation laws.

- Resolves shocks and rarefactions and is conservative

Unsplit algorithms in 2 dimensions



In one time step, conserved quantities can be transported across corners, so corner coupling must be taken into account.

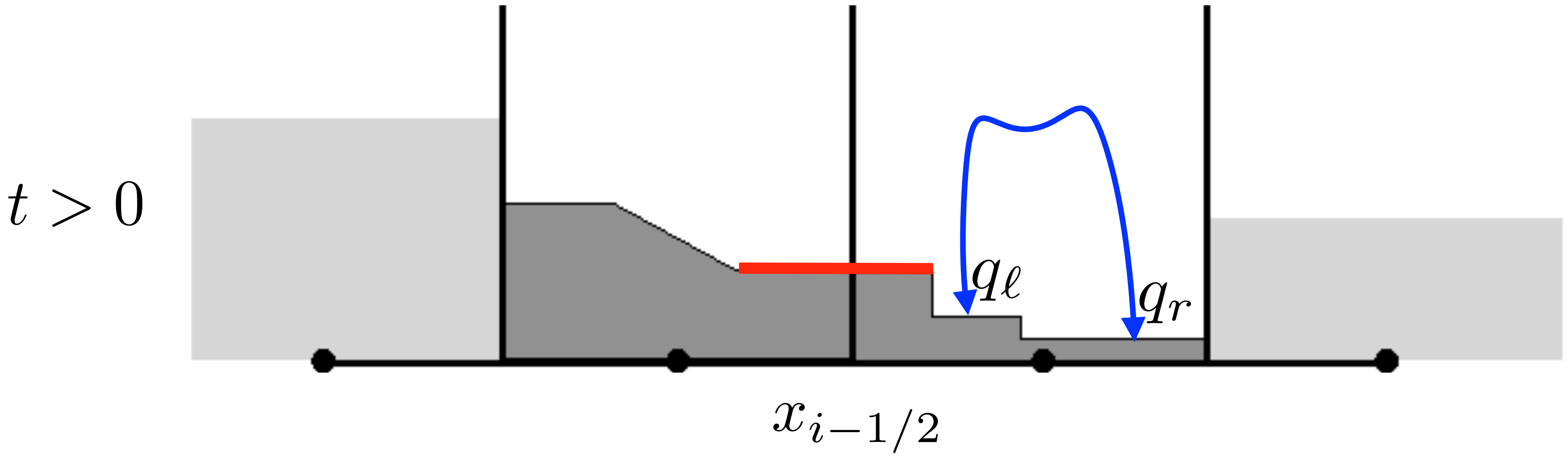
Approximate Riemann solvers

Conditions across shocks

Across shocks, the left and right states must satisfy the Rankine-Hugoniot conditions

$$f(q_r) - f(q_\ell) = s(q_r - q_\ell)$$

This leads to a nonlinear system for intermediate states.



Approximate Riemann solvers

$$q_t + f(q)_x = 0$$

We solve a linearized system at each cell interface, at each time step

$$q_t + f'(\hat{q})q_x = 0 \qquad \longleftrightarrow \qquad q_t + Aq_x = 0$$

for "Roe averaged" values $\widehat{q}$.

We use the eigenvalue/eigenvector decomposition of the flux Jacobian

$$A = R\Lambda R^{-1}$$

$$R = [r^1, r^2, \ldots r^m] \qquad\qquad \Lambda = \text{diag}(\lambda^1, \lambda^2, \ldots \lambda^m)$$

Eigenvalue decomposition

$$q_t + A\,q_x = 0 \quad \longrightarrow \quad q_t + R\Lambda R^{-1}\,q_x = 0$$

Define characteristic variables $\omega \in R^m$ as

$$\omega(x,t) = R^{-1}\,q(x,t), \qquad \omega(x,0) = R^{-1}\,q(x,0)$$

Characteristic equations decouple into m scalar equations :

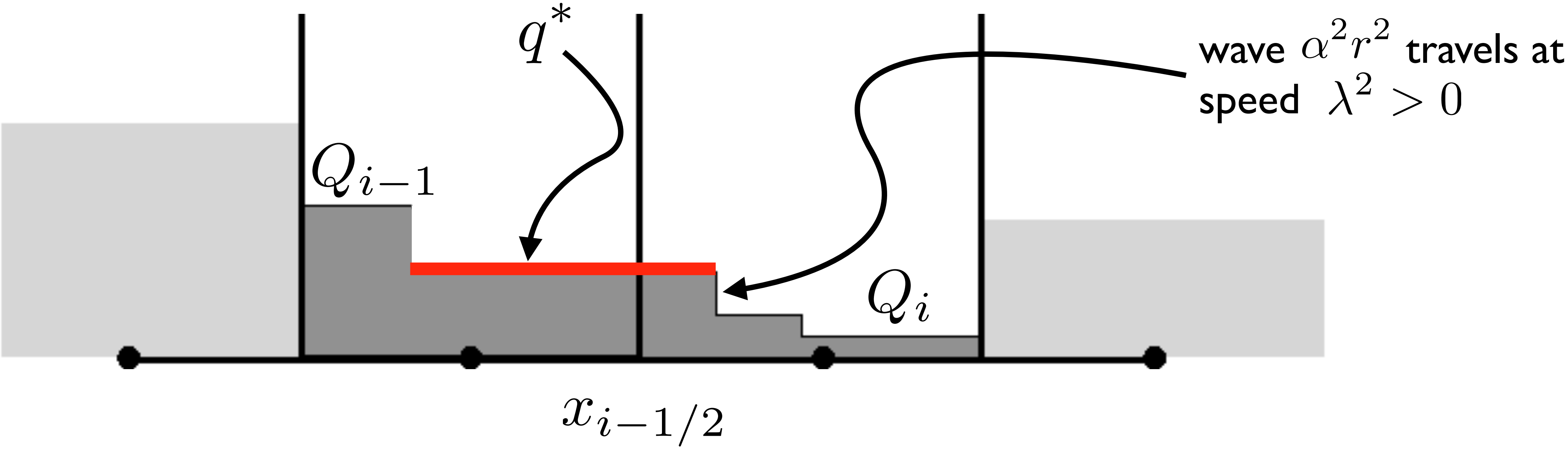
$$\omega_t^p + \lambda^p \omega_x^p = 0, \qquad p = 1, 2, \ldots, m$$

Solution for q(x,t) can be obtained from the solution to characteristic equations are given by

$$\omega^p(x,t) = \omega^p(x - \lambda^p t, 0)$$

Solution to linearized problem

For piecewise constant initial data, evolution looks like :



wave $\alpha^2 r^2$ travels at speed $\lambda^2 > 0$

Decompose jump in Q at the interface into waves :

$$q^* = Q_{i-1} + \alpha^1 r^1 = Q_i - \alpha^3 r^3 - \alpha^2 r^2$$

where $\alpha = R^{-1}(Q_i - Q_{i-1})$

Roe solvers

- Scheme is conservative if linearization is chosen correctly
- Much more efficient then solving the Riemann problem exactly (assuming analytic eigenvalue information is known).
- Gives accurate results, although may require a fix for transonic rarefaction waves
- Use "wave limiters" to suppress spurious oscillations

Wave propagation algorithms implemented in Clawpack (R. J. LeVeque) using "fluctuations". This approach handles both conservative and nonconservative terms in a uniform fashion.

www.clawpack.org

Multirate time stepping

Backup slides

Mappings and multiblock transforms

Mapped, multiblock domains

Assume each quadrilateral mesh cell can be approximated by a ruled surface.

$$\mathbf{S}(\xi,\eta) = \mathbf{a}_{00} + \mathbf{a}_{01}\xi + \mathbf{a}_{10}\eta + \mathbf{a}_{11}\xi\eta, \qquad 0 \le \xi,\eta \le 1$$

where $\mathbf{a}_{ij} \in R^{2\times 1}$ are computed from mesh cell vertices.

- Covariant basis vectors : $\tau_{(1)} \approx \mathbf{S}_\xi$ and $\tau_{(2)} \approx \mathbf{S}_\eta$
- Mesh cell areas : $\| \mathbf{S}_\xi \times \mathbf{S}_\eta \|$
- Edge normals and tangents
- Mappings are supplied as functions

```
[xp,yp,zp] = mapc2m(xc,yc)
```

Mapping contexts

```
fclaw2d_map_context_t* fclaw2d_map_new_squareddisk(const double alpha)
{
    fclaw2d_map_context_t *cont;
    cont = FCLAW_ALLOC_ZERO (fclaw2d_map_context_t, 1);
    cont->query = fclaw2d_map_query_squareddisk;
    cont->mapc2m = fclaw2d_map_c2m_squareddisk;

    cont->user_double[0] = alpha;
}
```

No need to rely on a fixed argument list; arguments specific to the mapping can be passed into the mapping routine.

```
c       ~~call mapc2m(xc,yc ,xd1,yd1,zd1)~~
        call fclaw2d_map_c2m(map_context,blockno,xc,yc,xd1,yd1,zd1)
```

```
static void
fclaw2d_map_c2m_squareddisk(fclaw2d_map_context_t *cont, int blockno,
                            double xc, double yc,
                            double *xp, double *yp, double *zp)
{
    double alpha = cont->user_double[0];
    mapc2m_squareddisk(&blockno,&xc,&yc,xp,yp,zp,&alpha);
}
```

Index transformations

```
c     # Exchange at left neighbor
      if (idir .eq. 0) then
         do j = 1,my
            do ibc = 1,mbc
               do mq = 1,meqn
                if (iface .eq. 0) then
c                    # Left face
                     i1 = 1-ibc
                     j1 = j
                  elseif (iface .eq. 1) then
c                    # Right face
                     i1 = mx+ibc
                     j1 = j
                  endif
                  call fclaw2d_transform_face(i1,j1,i2,j2,
     &                  transform_ptr)
                  qthis(mq,i1,j1) = qneighbor(mq,i2,j2)
               enddo
            enddo
         enddo
```

Loop over all ghost cells at left edge

This is the code used for every exchange at a left edge even if it is not a block boundary.

***transform_face** maps (i1,j1) ghost cell values to neighbor values, using information about block boundary orientations encoded in **transform_ptr**.*

Backup Slides

Misc