Practice #4 - Linear Algebra

1. Let $T(\mathbf{x}) = A\mathbf{x}$. Determine if $T(\mathbf{x})$ is one-to-one and if $T(\mathbf{x})$ is onto.

   (a) $A = \begin{bmatrix} 5 & 4 & -2 \\ 3 & -1 & 0 \end{bmatrix}$

   (b) $A = \begin{bmatrix} 2 & 8 & 4 \\ 3 & 2 & 3 \\ 1 & 14 & 5 \end{bmatrix}$

2. Perform the indicated computations, if possible, using the given matrices

$$A = \begin{bmatrix} 2 & 5 \\ 3 & -4 \end{bmatrix}, \qquad B = \begin{bmatrix} 3 & 1 \\ 4 & -5 \end{bmatrix}, \qquad C = \begin{bmatrix} 2 & 1 \\ 5 & 4 \\ 0 & -1 \end{bmatrix}, \qquad D = \begin{bmatrix} 2 & -2 & -3 \\ 0 & 3 & 1 \end{bmatrix}$$

   (a) $A + B$

   (b) AC

   (c) CB

   (d) A^2

   (e) $B - 3I_2$

(f) DB

(g) $C^T - D$

(h) $BA + DC$

3. Expand each of the given matrix expressions and combine as many terms as possible. Assume that all matrices are $n \times n$.

   (a) $(A + I)(A - I)$

   (b) $(A + I)(A^2 + A)$

   (c) $(A + B^2)(BA - A)$

   (d) $A(A + B) + B(B - A)$

4. Why are the following matrix equations false?

   (a) $(A + B)^2 = A^2 + 2AB + B^2$

   (b) $A^2 - B^2 = (A - B)(A + B)$

5. If A is a symmetric matrix, show that $A + A^T$ is also symmetric.