

## Practice #5 - Matrix operations

1. Matrices A, B, C and D have dimensions given by :

- $A$  is  $5 \times 7$
- $B$  is  $4 \times 5$
- $C$  is  $3 \times 4$
- $D$  is  $7 \times 1$
- $E$  is  $1 \times 4$

- (a) Indicate how many rows and how many columns each matrix has.
- (b) How many matrix products can you form with these matrices? Indicate the dimensions of the resulting products. Include transpose operators and matrix powers.
- (c) Can you construct a  $4 \times 4$  square matrix from the above?
- (d) Can you construct a  $5 \times 5$  square matrix from the above?
- (e) Can you produce a scalar (a  $1 \times 1$  matrix) from the above?
- (f) Construct a matrix that can be raised to a power.
- (g) Create a matrix product with the most possible number of entries?
- (h) Suppose  $F = BA$ . Describe how you would compute entry  $f_{26}$  of  $F$ .

2. A matrix  $A$  is given by

$$A = \begin{bmatrix} 3 & 0 & 5 & 3 \\ 1 & 2 & -1 & 2 \end{bmatrix}$$

Find two identity matrices that satisfy  $AI = A$  and  $IA = A$

3. Suppose  $A$  and  $B$  are  $n \times n$  matrices and  $BA = I$ . What is a solution to  $A\mathbf{x} = \mathbf{b}$ ? Assume that  $\mathbf{x}$  and  $\mathbf{b}$  are  $n \times 1$  vectors.

4. Expand each of the given matrix expressions and combine as many terms as possible. Assume that all matrices are  $n \times n$ .

(a)  $(A + I)(A - I)$

(b)  $(A + I)(A^2 + A)$

(c)  $(A + B^2)(BA - A)$

(d)  $A(A + B) + B(B - A)$

5. Why are the following matrix equations false?

(a)  $(A + B)^2 = A^2 + 2AB + B^2$

(b)  $A^2 - B^2 = (A - B)(A + B)$

6. If  $A$  is a symmetric matrix, show that  $A + A^T$  is also symmetric.