Practice #1 - Linear Independence

1. What is the difference between these two sets of vectors?

$$\left\{\begin{bmatrix} 3 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ 4 \end{bmatrix}\right\} \qquad \text{and} \qquad \left\{\begin{bmatrix} 1 \\ 3 \end{bmatrix}, \begin{bmatrix} -2 \\ -6 \end{bmatrix}\right\}$$

   Based on your observation, formulate a mathematical definition of "linear independence".

2. According to your definition, is the following set "linearly independent"?

$$\left\{\begin{bmatrix} 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 6 \\ -5 \end{bmatrix}, \begin{bmatrix} 1 \\ 4 \end{bmatrix}\right\}$$

   (a) Formulate a vector equation involving all three vectors and solve the equation to determine if the vectors are independent.

3. Determine if the following set is linearly independent.

$$\left\{ \begin{bmatrix} 4 \\ 0 \\ -3 \end{bmatrix}, \begin{bmatrix} -2 \\ -1 \\ 5 \end{bmatrix}, \begin{bmatrix} -8 \\ 2 \\ -19 \end{bmatrix} \right\}$$

   (a) Formulate three different vector equations for this problem. Can they all be correct? How do you decide which one is "correct"?

   (b) Is there a single "universal" equation that will cover all cases? **Hint** : This "universal" equation is a homogeneous equation.

   (c) Solve a single "universal" vector equation to determine if the vectors are linearly independent.

4. Write down the steps in determining whether a set of vectors of linearly independent. **Hint:** There are three steps!

   (a) Formulate ...

   (b) Solve ...

   (c) Decide ...