

Practice #7 - Linear Algebra

1. Use the "row method" to find a basis for set spanned by the following set of vectors.

$$\mathbf{u}_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}, \quad \mathbf{u}_2 = \begin{bmatrix} 2 \\ 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{u}_3 = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{u}_4 = \begin{bmatrix} 3 \\ 1 \\ -1 \\ 3 \end{bmatrix}$$

What is the dimension of the space spanned by this set?

2. Use the "column method" to find a basis for set spanned by the following set of vectors.

$$\mathbf{u}_1 = \begin{bmatrix} 1 \\ 0 \\ 3 \\ 1 \end{bmatrix}, \quad \mathbf{u}_2 = \begin{bmatrix} 4 \\ 2 \\ 13 \\ 4 \end{bmatrix}, \quad \mathbf{u}_3 = \begin{bmatrix} 2 \\ 1 \\ 6 \\ 3 \end{bmatrix}, \quad \mathbf{u}_4 = \begin{bmatrix} -1 \\ 1 \\ -2 \\ -2 \end{bmatrix}$$

What is the dimension of the space spanned by this set?

3. Find a basis for the column space, the row space and the null space of A . Verify that the Rank-Nullity Theorem holds.

$$A = \begin{bmatrix} 1 & 2 & 0 & -1 \\ -2 & -3 & -1 & 4 \\ 1 & 4 & -2 & 4 \\ 2 & 2 & 2 & -4 \end{bmatrix} \sim B = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

4. Find a basis for the column space, the row space and the null space of A . Verify that the Rank-Nullity Theorem holds.

$$A = \begin{bmatrix} 1 & 4 & -1 & 1 \\ 3 & 11 & -1 & 4 \\ 1 & 5 & 2 & 3 \\ 2 & 8 & -2 & 2 \end{bmatrix}$$

5. Suppose that A is a 6×8 matrix. If the dimension of the row space of A is 5, what is the dimension of the column space of A ?

6. Suppose that A is a 9×7 matrix. If the dimension of $\text{col}(A)$ is 5, what is the dimension of $\text{row}(A)$?

7. Suppose that A is a 9×7 matrix that has an echelon form with one zero row. Find the dimensions of the column space of A , the row space of A and the null space of A .

8. A 5×13 matrix A has a null space of dimension 10. What is the rank of A ?

9. Suppose that A is a 6×11 matrix and that $T(\mathbf{x}) = A\mathbf{x}$. If $\text{nullity}(A) = 7$, what is the dimension of the range of T ?

10. Suppose that A is a 17×12 matrix and that $T(\mathbf{x}) = A\mathbf{x}$. If $\text{rank}(A) = 8$, what is the dimension of the kernel of T ?

11. Suppose that A is a 5×13 matrix and that $T(\mathbf{x}) = A\mathbf{x}$. If T is onto, then what is the dimension of the null space of A ?

12. (True/False) If A is a square matrix, then $\text{row}(A) = \text{col}(A)$.

13. (True/False) The rank of A cannot exceed the number of rows of A .

14. (True/False) If \mathbf{y} is a solution to $A\mathbf{x} = \mathbf{b}$, then \mathbf{y} is in $\text{row}(A)$.