Review for Exam #1 - Linear Algebra

The exam will cover material through Chapter 2. You are only responsible for the Chapters we covered.

1. **Concepts.** Know the difference between a *linear system*, a *vector equation*, and a *matrix equation.*

   Example :

   (a) Write the following linear system as a vector equation and a matrix equation.

$$\begin{aligned} 2x_1 + 3x_5 &= 4 \\ -x_2 - x_4 &= 0 \\ x_3 - x_5 &= -1 \end{aligned} \tag{1}$$

2. **Solve linear general linear systems**. Be sure you know how to solve a general linear system (written in any form) using row-reduction. Be able to express the solution in terms of free variables, if necessary.

   Example : Solve the following matrix equation $A\mathbf{x} = \mathbf{b}$, where

$$A = \begin{bmatrix} 2 & -6 & -1 & 8 \\ 1 & -3 & -1 & 6 \\ -1 & 3 & -1 & 2 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} 7 \\ 6 \\ 4 \end{bmatrix}$$

3. **Row reduction.** Know how to row-reduce a matrix so that you can answer the following questions:

   (a) Are the columns of the matrix linearly independent?

   (b) Do the columns of the matrix span R^n?

   (c) Does a matrix equation have solutions for any $\mathbf{b}$?

   Example : Answer the above questions using the matrices from Problem 1 and Problem 2.

4. **Linear Independence** Consider the following three vectors.

$$\begin{bmatrix} 1 \\ 7 \\ 8 \\ 4 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \\ 5 \\ 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ -2 \\ 0 \end{bmatrix}$$

   (a) Solve a homogeneous linear system to determine if one of the vectors is in the span of the others.

   (b) If possible, and without doing any more work, write the second vector in terms of the first and third vector.

   For a problem like this, you must show all of your work. You will only get partial credit if you somehow manage to guess the answer.

5. **Theory**. Be prepared for some true/false questions or other short answer problems. If the answers are False, be prepared to explain why.

   Problem. A matrix A has 4 rows and 2 columns. The matrix equation $A\mathbf{x} = \mathbf{0}$ has exactly one solution. Answer the following questions.

   (a) The columns must be linearly dependent (T/F)

   (b) The columns span $\mathcal{R}^2$. (T/F)

   (c) Let $\mathbf{b}$ be a vector of all 1s. Then the equation $A\mathbf{x}$ has an infinite number of solutions. (T/F)

   (d) Let $\mathbf{x}$ be the vector $(3, 4)$ and let $\mathbf{u} = A\mathbf{x}$. Then the set $\{\mathbf{u}, \mathbf{a}_1, \mathbf{a}_2\}$ is linearly independent. (T/F)

   (e) There are non-zero values c_1 and c_2 such that $c_1\mathbf{a}_1 + c_2\mathbf{a}_2 = 0$. (T/F)

   (f) The reduced row echelon form of the matrix will have exactly 2 pivots. (T/F)

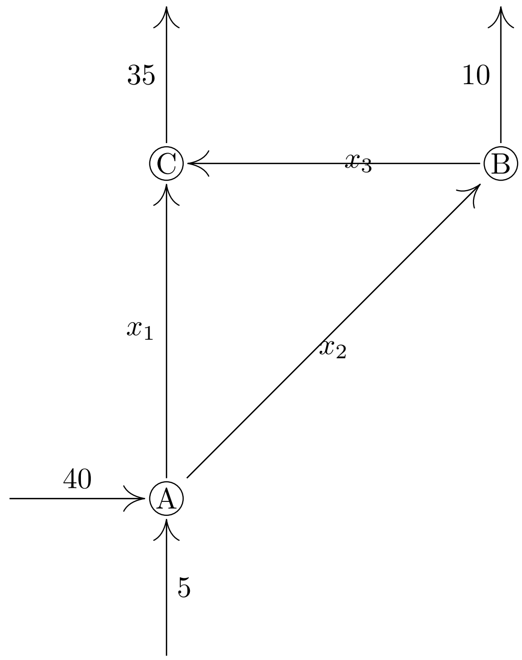

Figure 1: Figure used Problem 6a

6. **Applications.** You will have one or two application problems to solve. You must show all of your work, even if you can solve the problem by inspection.

   (a) **Traffic flow.** For this problem, refer to Figure 1. What is the flow of traffic along routes x_1, x_2 and x_3? Provide the most general answer that you can.

   (b) **Chemical reactions** Sodium and water react to form sodium hydroxide (lye) and hydrogen. Balance the chemical equation

$$\mathrm{Na} + \mathrm{H_2O} \to \mathrm{NaOH} + \mathrm{H}^2$$