Homework # 4

Math 427/527

Note : Math 427 students may do the Math 527 questions for extra credit. You may work in pairs on this assignment, but pairs can only be two 427 students or two 527 students but not mixed pairs.

All plots must have axes labels, and a title. Also, be sure to use appropriate axis limits for each plot. Make your plots interesting!

1. Use Stokes Theorem to evaluate the surface integral

$$\iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n} \, dA$$

for given $\mathbf{F}$ and S.

   (a) $\mathbf{F} = [z^2, -x^2, 0]$, S is the rectangle with vertices $(0,0,0)$, $(1,0,0)$, $(0,4,4)$, $(1,4,4)$.

   (b) $\mathbf{F} = [e^y, e^z, e^x]$, $S : z = x^2$, $(0 \le x \le 2, \quad 0 \le y \le 1)$

2. Stokes Theorem is not applicable in all cases. Evaluate

$$\oint_C \mathbf{F} \cdot \mathbf{r}'(s) ds$$

for $\mathbf{F} = (x^2 + y^2)^{-1}[-y, x]$ where $C : x^2 + y^2 = 1$, $z = 0$, oriented clockwise. Why doesn't Stoke's Theorem work here?

3. For the following questions, use the vector field $\mathbf{F} = (e^y, e^z, e^x)$.

   (a) Use the Matlab code from the course website to plot $\nabla \times \mathbf{F}$ over the surface $x^2 + y^2 \le 1$, $z = 0$. When you run this code for "example = 1", you will also see the following printed out :

```
Surface integral    -869.1697836360004885
   Line integral     -22.6063641596993925
      Difference              8.4656e+02
```

   These are the results of computing the surface integral and line integral from Stoke's Theorem, and so they should be approximately equal. Find two bugs in the code so that you get the results

```
Surface integral      -3.5504553921995265
   Line integral      -3.5509993784243603
      Difference              5.4399e-04
```

   (b) After you have fixed the bugs, replace the flat disk used above with the surface $z = a(x^2 + y^2)$, $x^2 + y^2 \le 1$ for any real number a. Explain what key feature of Stoke's Theorem this problem demonstrates. Test your code for a few different values of a and submit your results. Also, submit a plot for $a = -1$.

4. (**Math 527.**) For the following questions, use the vector field $\mathbf{F} = [y^2, x^2, z^2]$.

   (a) Verify Stoke's Theorem for vector field $\mathbf{F}$ over the surface $S : (u, u^2, v)$, $0 \le u \le 2$, $-2 \le v \le 2$.

   (b) Complete "TODO" sections for Example 2 of the code on the course website. You should see the the results :

```
Surface integral       0.0000000000000000
   Line integral      -0.0000000000000115
      Difference              1.1458e-14
```

Submit the plot you get, along with your completed code.

(c) What is the fallacy in the following argument : "Since the line integral around the close curve is zero, the vector field **F** must be conservative" ?

5. (**Fourier Analysis**). Let $f(x) = x^2$ on $[-\pi, \pi]$, and $f(x + 2\pi) = f(x)$.

(a) Graph $f(x)$ over the interval $[-3\pi, 3\pi]$.

(b) Find the Fourier series for $f(x)$.

(c) Sketch or graph the partial sums of the series up to terms including $\cos(5x)$ and $\sin(5x)$.

(d) (**Math 527**). Does this series converge? Why or why not?