

## Practice #9 - Linear Algebra

1. Compute the determinant of the following matrices.

(a)

$$A = [-5]$$

$$\det(A) = \underline{\hspace{2cm}}.$$

(b)

$$A = \begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix}$$

$$\det(A) = \underline{\hspace{2cm}}.$$

(c)

$$A = \begin{bmatrix} 2 & 2 & 3 \\ -1 & 4 & 1 \\ 3 & 1 & -2 \end{bmatrix}$$

$$\det(A) = \underline{\hspace{2cm}}.$$

2. Find the determinant of a  $5 \times 5$  diagonal matrix whose diagonal entries are 1, 2, 3, 4, 5.

3. Find the determinant of a  $5 \times 5$  upper triangular matrix whose diagonal entries are all ones.

4. Find the determinant of a  $5 \times 5$  lower triangular matrix whose diagonal entries are all ones.

5. Find the determinant of an identity matrix  $I_5$  in which row 2 and row 3 have been swapped.

6. Find the determinant of an identity matrix  $I_5$  that has its rows in reverse order, so that the diagonal goes from the lower right to the upper left.

7. Find the indicated quantity for the following matrix  $A$

$$A = \begin{bmatrix} 3 & -4 & 0 & 5 \\ 2 & 1 & -7 & 1 \\ 0 & -3 & 2 & 2 \\ 5 & 8 & -2 & -1 \end{bmatrix}$$

(a)  $M_{23} =$

(b) the minor of  $a_{31}$

(c)  $C_{43}$

(d) A co-factor expansion about row 3.

8. Let  $I_5$  be the  $5 \times 5$  identity matrix. What is  $\det(-2I_5)$ ?
9. Suppose  $\det(A) = 6$  and  $\det(AB) = 21$ . What is the determinant of  $B$ ?
10. Let  $A$  be a  $5 \times 5$  matrix whose determinant is 35. Now swap rows 1 and 2 of  $A$  to get a matrix  $B$ . What is the determinant of the matrix  $B$ ? (Hint : Try this out on the matrix in Problem 1b.)
11. Let  $A$  be a square matrix and let  $B$  be the echelon form of  $A$ . What do you guess is the relationship between  $\det(A)$  and  $\det(B)$ .

- Suppose that  $A$  has an  $LU$  factorization, so that  $A = LU$ , where  $L$  is a lower triangular matrix with all ones on the diagonal and  $U$  is an upper triangular matrix. What would be an easy way to compute  $\det(A)$  using this factorization?